Extra Exercises for Chapter 18.  
Cyclical Behavior in a Model of a Private College

These exercises build upon the model of a private college from the extra exercises for chapter 7. The model showed S-Shaped growth in the number of students enrolled in the college. The previous model assumed that a change in the student-faculty ratio leads instantly to a change in the students' willingness to nominate new students for admission. Now consider the model in Figure 1. It assumes that there is a delay in the students' behavior. It uses a SMTH1, a first order smooth function, to calculate the delayed value of the ratio. (The first order smooth is explained in chapter 14. It makes sense for this example if some of the students respond to the higher ratios almost immediately.) The delayed value of the ratio is then used to find the students' nomination rate (based on the same nonlinear relationship in the extra exercises for chapter 7). The maximum nomination rate allowed by the college is 4 nominees per student per year. This maximum will not be a factor since the students nomination rate is not inclined to exceed 2 nominees per student per year.

![Diagram of a college with a delay in the students' reaction to the student/faculty ratio.]

Figure 1. Model of a college with a delay in the students’ reaction to the student/faculty ratio.

Build this model to show the impact of the delay in the students’ delayed reaction to the student-faculty ratio. Assume that all parameters are the same as the extra exercise from chapter 7. Set the lag time to 1 year.

1. Expectations Before Simulating:

Before running the model (and before looking at the next page), write down your expectations for the new college. Do you believe the model will show S shaped growth? Do you think the college will eventually end up with the same number of students as in the extra exercise for chapter 7? Do you expect oscillations? Will they dampen out over time? Or will you see oscillations that continue indefinitely?
2. Simulate the New College Model

Simulate the new model over 20 years with a DT of 0.25 years, and check that you get the behavior shown in Figure 2. The number of existing students is simulated to grow to over 8,000 within the first four years. It then drops rapidly reaching a low of fewer than 3,000 by the 7th year. The simulation then reveals that the college would experience damped oscillations and eventually arrive at an enrollment of around 3,500.

![Figure 2. Simulated behavior of college model over 20 years.](image)

3. Are These Results Numerically Accurate?

Do you think a DT of 0.25 years is sufficiently small for numerical accuracy? To test your instincts, cut DT in half and repeat the simulation. Do you get the same results? If not, cut DT in half again, and repeat the simulation again. You will eventually arrive at accurate results. Does the accurate simulation give a final enrollment around 3,500 (as in Figure 2)? Do the oscillations in the accurate simulation have a smaller amplitude than the oscillations in Figure 2?

4. Avoid the Wild Beginning

Suppose the college wishes to avoid the "roller coaster" behavior shown in first ten years of Figure 2. Assume the only point of intervention is the maximum number of new students that existing students may nominate per year. The current maximum is 4 nominees per student year. What limit might you impose to allow the college to avoid the “roller coaster” in the first ten years? Can you keep the oscillations from getting too large and still build the student size to 3,500 in 5 or 10 years? Document your results with a comparative time graph showing the number of existing students in the base case simulation (shown above) and in the simulation with your suggested limit on nominees.