Chapter 4. Accumulating the Flows

System dynamics models are constructed as a combination of stocks and flows and then simulated on the computer. The simulation results are generated in a step-by-step fashion by accumulating the effect of the flows. This chapter illustrates the process with three examples. It then describes what we must do to ensure that the computer simulations are accurate.

Numerical Accumulation of Carbon Dioxide in the Atmosphere

Let’s start with the portion of the global carbon cycle shown in Fig. 4.1. Chapter 23 explains that the growing emissions of carbon dioxide (CO2) could cause the CO2 in the atmosphere to double in this century. We’ll demonstrate how this can happen by the pencil-and-paper calculation in Table 4.1. The model adds CO2 to the atmosphere from anthropogenic (man-made) emissions. CO2 is removed from the atmosphere by the net exchanges between the terrestrial and oceanic systems. The net removals are represented by two flows with long names to remind us of the assumptions adopted for this example. CO2 in the atmosphere is a stock measured in Gigatons of Carbon, abbreviated GTC. (The C stands for the C in CO2). The starting value is 750 GTC in the year 2000. Anthropogenic emissions are due mainly to the combustion of fossil fuels. We start with emissions of 6.4 GTC/year for the first decade.

<table>
<thead>
<tr>
<th>Year</th>
<th>Anthropogenic Emissions (GTC/yr)</th>
<th>Net Removal to Biomass &amp; Soils (GTC/yr)</th>
<th>Net Removal to Oceans (GTC/yr)</th>
<th>CO2 in Atmosphere (GTC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td></td>
<td></td>
<td></td>
<td>750.0</td>
</tr>
<tr>
<td>2000-2010</td>
<td>6.4</td>
<td>1.0</td>
<td>2.0</td>
<td>784.0</td>
</tr>
<tr>
<td>2010-2020</td>
<td>7.0</td>
<td>1.3</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2020-2030</td>
<td>8.0</td>
<td>1.6</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2030-2040</td>
<td>9.0</td>
<td>1.8</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2040-2050</td>
<td>10.0</td>
<td>1.9</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2050-2060</td>
<td>11.3</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2060-2070</td>
<td>12.7</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2070-2080</td>
<td>14.2</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2080-2090</td>
<td>16.0</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2090-2010</td>
<td>18.0</td>
<td>2.0</td>
<td>2.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1. Complete this table to find the CO2 in the atmosphere at the end of the century.

Emissions increase each decade due to the growth in fossil fuel combustion. The net removal by the terrestrial system is 1 GTC/year during the first decade. Some climate modelers predict that net removal will grow over time, so net removal increases in each decade during the first half of the century. Net removal remains at 2 GTC/year for the second half of the century. Net removal by the oceans is fixed at 2 GTC/year for the entire century.
The table is designed for a step-by-step calculation of the cumulative effects of the three flows. We will proceed one decade at a time. Anthropogenic emissions are growing continuously over time due to the increase in the global combustion of fossil fuels, but the table shows average values for each decade. The first row shows the accumulation during the decade from 2000 to 2010. The average value of emissions is 6.4 GTC/year, so 64 GTC would be added to the stock during this decade. The net removal is 3 GTC/year, so 30 GTC would be removed from the atmosphere during this decade. The new value of the stock would be 750 + 64 – 30 which is 784.

I’ve entered this value to get you started. To appreciate accumulation, you should complete the rest of the table. If you do the calculations properly, you will arrive at 1,500 GTC of CO2 in the atmosphere at the end of the table. Atmospheric CO2 will double in one century. You’ll read in chapter 23 that a doubling of CO2 is predicted by many of the climate modeling teams around the world, and the doubling can arise from the general assumptions shown here. So this is an important projection which we can anticipate by simply accumulating the flows in a step-by-step manner. This is what we mean by “numerical simulation.”

**Numerical Simulation and the Step Size**

Stella and Vensim simulate models in a step-by-step manner, similar to the calculations in Table 4.1. The calculations didn’t take long because you only needed to update the stock very decade. The “step-size” of the calculations was ten years, so you performed the calculations ten times. If I had provided values for every year, you could follow the same approach with a one-year step size. Your calculations would be done in 100 steps. If the flows were changing during the course of a single year, a smaller step-size would be appropriate. If the step-size were a quarter of a year, your calculations would have been completed in 400 steps. Stella and Vensim perform such calculations quickly and accurately. This process is called numerical simulation because the results are found by making simple, numerical updates in the stocks. Once we have specified the model, our job is easy. We pick the step-size and click the “run” button.

**Visual Accumulation of Water in a Reservoir**

Fig. 4.2 shows a model to accumulate water in a reservoir. The storage is measured in thousands of acre-feet, which we abbreviate as KAF. The model runs in months, so the flows are measured in KAF/month. The purpose of the reservoir is to deliver a constant outflow of 5 KAF/month.
Chapter 4. Accumulating the Flows

The inflow is highly variable, as shown in Fig. 4.3. The inflow peaks in the 5th month, and it dips to the lowest point in the 8th month. The total inflow over the entire year is 60 KAF, so the average inflow is 5 KAF/month. This is the same as the outflow, so we know the water stored in the reservoir at the end of the year will be the same as at the start of the year. But the reservoir will gain and lose storage within the year. I haven’t given the starting value of the storage, and the graph does not give the exact numbers for the inflows. So you can not perform a numerical calculation like the CO2 example. But you should be able to visualize how the reservoir storage will rise and fall over time. Give this a try and answer two questions about water storage:

- When will reservoir hold the most water?
- When will it hold the least water?

The answers may be found on the BWeb. Previous students’ work on this and related exercises have revealed a wide disparity in their ability to think about the accumulation of flows over time. For one reason or another, many students misread the cumulative impact of the flows. One of the most frequent tendencies is to overstate the stock’s responsiveness to the changing flows. On the other hand, some students do the accumulations correctly the first time; many others can teach themselves about accumulation with practice. You can practice with the exercises at the end of the chapter. They will help you improve your understanding of accumulation. With better understanding, you’ll be in a better position to anticipate the results of computer simulations.

**Computer Simulation of Population Growth**

The final example is a population model in Fig. 4.4. It starts with 10 million people. The growth rate is 20%/yr, so the population will grow will double to 20 million people in just 3.5 years (see appendix B). We will simulate the model for 5 years with DT set to 1 year. Stella uses “DT” for the step-size of the numerical simulation. (DT stands for delta time, the increment of time between steps in the simulation.) Our job is to select the value of DT to give an accurate simulation. The population is growing continuously, so we would obtain the most accurate results with an small value of DT. DT can be set to a fraction of a year, so we might pick 1/8th of a year to give accurate results. A five year simulation would
require 40 steps. To check the accuracy, we could set the DT to $1/16^{th}$ of a year, and the new simulation will take 80 steps. We then compare the simulations. If we see essentially the same results, we know the original DT is sufficiently small to give numerical accuracy.

Fig. 4.5 shows the accuracy in simulations with DT set to 1 year (top graph) and $1/2$ year (lower graph). The accurate result is labeled population in both graphs. The stair-case patterns show the numerical values that are updated with each DT. Each graph has a small circle to mark the spot with 20 million people and 3.5 years. (These are Stella’s information buttons. They remain in view if the graph is pinned to the diagram.) It’s clear that the population graphs are on target, but the step-by-step pattern is lagging behind. The errors are larger in the top graph, and they are growing as the simulation proceeds. The errors are smaller in the lower graph, but these errors are also growing larger over time. If you repeat these tests, you’ll find that you get increasingly accurate results with smaller values of DT. In this example, there are vanishing small errors if DT is $1/16^{th}$ of a year.

**Advice on DT**

The Stella software asks us to specify the units of time and to set the value of DT. The default value of DT is 0.25, with the units for DT the same as the units of time. In the population model, the default DT is 0.25 years. This turns out to be a good selection for most models, but it is not sufficiently small for the rapidly growing population in Fig. 4.5. With 20% annual growth in population, we need a smaller DT to keep pace with the upward trend. The same reasoning applies if the population were on a steep down trend. If the change is rapid, we need to update the stocks frequently to keep up with the trends.

The easiest way to pick the value of DT is to draw a graph of the expected pattern over time. (We should have such a graph on hand, as you know from the 2nd step of
modeling; it calls for a “reference mode” graph to help us be specific about the dynamic problem that is the focus of the modeling.) To pick a value of DT, imagine a stair case pattern that follows the ups and downs in your graph. The width of the step in the stair case is a good guess at DT. You simply pick a sufficiently small DT so the stair case pattern follows your graph closely. Run the model with this value of DT. Then cut DT in half and run the model again. If you get essentially the same results, you have numerically accurate results.

<table>
<thead>
<tr>
<th>How close is close enough?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joe understands that the simulations will be increasingly accurate as he cuts DT in half. But he isn’t sure about the phrase “essentially the same results.” Do all the results have to be close or just the results at the end of the simulation? And how close is close enough? Do we call the results accurate if they are within 1% of the results in the previous simulation? This is one of those questions that can’t be answered in the classroom, but the answer will be clear when you deal with a real problem. You will have specified a reference mode and will be striving for a general understanding of the problem. With this context, you will have no problem deciding whether the new simulation is “essentially the same” as the previous simulation with the larger DT. As you make this judgment, remember that we are modeling for general understanding of dynamic patterns. We are not making point predictions or forecasts of the future state of the system.</td>
</tr>
</tbody>
</table>

**Final Suggestion**

The final guideline for setting DT is to remember that:

*DT has no counterpart in the real world.*

In other words, we should not be tempted to set the value of DT to match some time interval in the simulated system. DT is the step-size used in numerical integration; it has nothing to do with a time-interval in the real world. When simulating automobile sales, for example, do not be tempted to set DT to 0.25 years if you happen to have quarterly data on sales. If you are simulating a water shed with monthly data on flows, do not feel that DT should be 1 month. DT must be set sufficiently small to ensure numerical accuracy. And to check the accuracy, we cut the value in half and repeat the simulation. We can not follow these rules if we make the mistake of associating DT with some time interval in the real world.

The temptation to match DT to a real-world time interval occurs among those who envision the world advancing in discrete steps. (Perhaps crops are planted each spring, and the market prices are evaluated when they are harvested in the following fall.) If you want a model to match this vision of the world, turn to different modeling methods (BWeb). System dynamics models are simulated on a continuous basis, with time advancing one small step at a time. The temptation to set DT to match a real time interval can also arise when students see the system responding abruptly at a particular time each year. An example could be the sudden appearance of the Spring Chinook salmon in the Columbia each spring. We do not set DT to 1 year to represent such a pattern. It makes much more sense to use conveyor stocks, as explained in chapters 14 and 15.
Conclusion and Discussion

The examples in this chapter require a few hundred steps for the numerical simulations. The CO2 model in Table 4.1 requires only 100 steps if DT were 1 year. It would require 400 steps if DT were 1/4th of a year. However, suppose the atmospheric CO2 were changing much more rapidly, and we changed DT to 1/16th of a year. We would now need 1,600 steps. I recommend that we think twice about simulations that require more than 1,000 steps. The BWeb discusses ways to avoid such simulations. It explains that the standard integration method is first-order integration, sometimes called Euler integration. The BWeb discusses higher-order integration methods to obtain faster simulations. As a general rule, simulations should not require us to resort to higher-order methods. A better approach is to think carefully about the high-turnover stocks that are forcing us to select a small value of DT (as explained in chapter 17).

We'll talk more about DT and numerical accuracy later in the book. But at this point, you know enough to generate numerically accurate simulations. Pick a reasonable value of DT and simulate the model. Cut the DT in half and simulate again. If you get the same results, you have numerical accuracy. Remember that you should not confuse numerical accuracy with model realism. Numerical accuracy means the simulation results are an accurate calculation based on your assumptions; it has nothing to do with the realism or your assumptions or your simulation results.

Exercises

Exercise 4-1. Bees in the Hive: Fig. 4.6 shows the flow of bees in and out of a hive. Fig. 4.7 shows a time graph of the two flows over a 30 minute period.

Questions on the flows: During which minute did the most bees enter the hive? During which minute did the most bees leave the hive? Questions on the stock: During which minute were the most bees in the hive? During which minute were the fewest bees in the hive?
Exercise 4-2. Population Sketch: Fig. 4.8 shows a model of an animal population subject to births and deaths. There are 100 animals at the start of the simulation.

Deaths are fixed at 50 animals/year. The births are 100 animals/year when time is 0.0 years. By the time reaches 2.0 years, the births have fallen to 50 animals/year. The births and deaths are occurring continuously over time. The dot marks the starting population at 100. Draw the population curve for the rest of the time interval.

Exercise 4-3. Population Simulation: Construct the model in Fig. 4.8. The ~ for births indicates that a graphical function has been used to make the births change over time. Open the births flow and type time. Then click the button for become graphical function, and you will see a graphical box (similar to the box in Fig. 2.18). Enter values for births to match the values in Fig. 4.9. Then simulate the model for 16 years with DT = 0.25 years. Does the population match your sketch?

Exercise 4-4. Stella’s Sketchable Graph: Sketchable graphs are a device to encourage us to think about the likely results before we run the model. Add a new graph to the previous model and click on the sketchable option. This will create a graph of a selected variable that may be compared to your sketch of the expected behavior. Select population to appear on a scale from 0 to 200. Then drag the cursor across the graph to give a rough approximation to the pencil sketch from exercise 4-2. Then simulate the model. You will see the results along with your sketch.