Extra Exercises for Chapter 6. Equilibrium Diagrams

1. Urban Dynamics:

Figure 1 shows an equilibrium diagram adapted from Gerald Barney's (1974, p. 37) explanation of the dynamic equilibrium in the flows of workers into and out of an urban area. This diagram portrays a city of just over 1 million workers in dynamic equilibrium. (These particular flows appeared in a computer simulation of "revival" policies in Forrester's *Urban Dynamics*.) The stocks represent thousands of workers in three categories of employment. The flows are measured in thousands of workers per year. Recall that a flow with two arrowheads is a biflow -- a flow that may be positive or negative. Also, recall that that the white arrowhead portrays the positive direction. Since the urban system is in dynamic equilibrium, you should be able to answer the ?? in the diagram:

What is the value of the "net departures of managers"? ______________________

What is the value of the "net departures of laborers"? ________________________

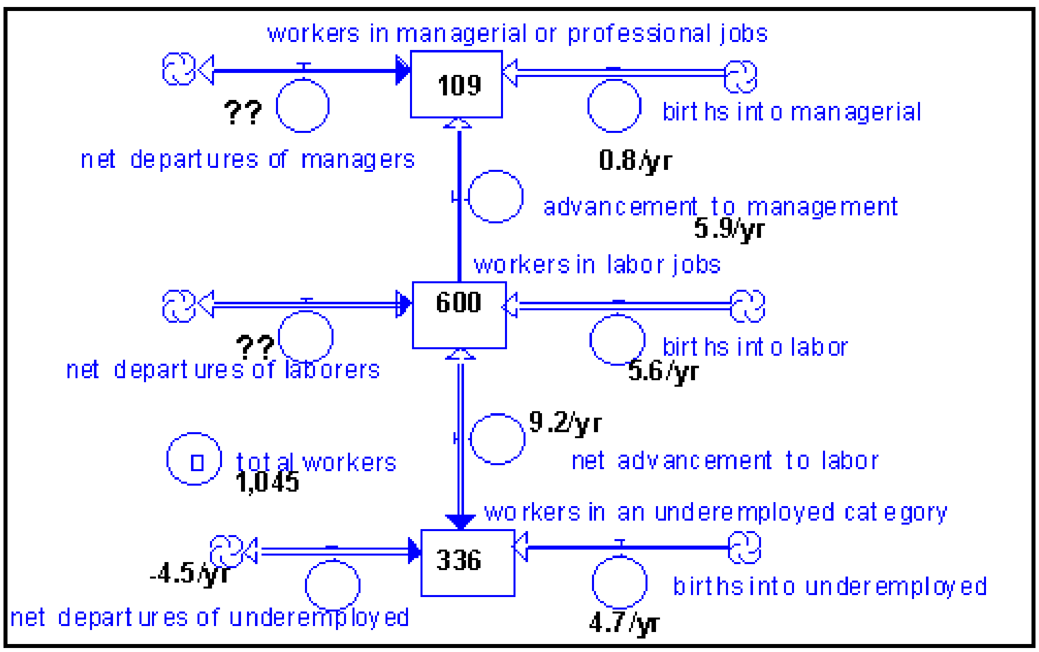

Figure 1. Equilibrium diagram with missing values

2. The Dutch Health Care System

Figure 2 is an equilibrium diagram adapted from Luc Verburgh's (19794 p. 139) model of the hospital sector of the Dutch health care system. The stock at the top of the diagram shows 6,428 people waiting for admission to a hospital. This stock is increased by referrals from medical specialists which are set at 803 people/week. The stock is drained when the patients are admitted to the hospital. The average time waiting for admission is influenced by the occupancy of the hospital.

What is the flow of "patients admitted"? ____________________
What is the "average waiting time for admission"? ____________________________

(Note: Some students say these values cannot be determined unless you are told the equations. This model is like others in the book. It relies on "friendly algebra," so you should be able to guess the equations from the names of the variables.)

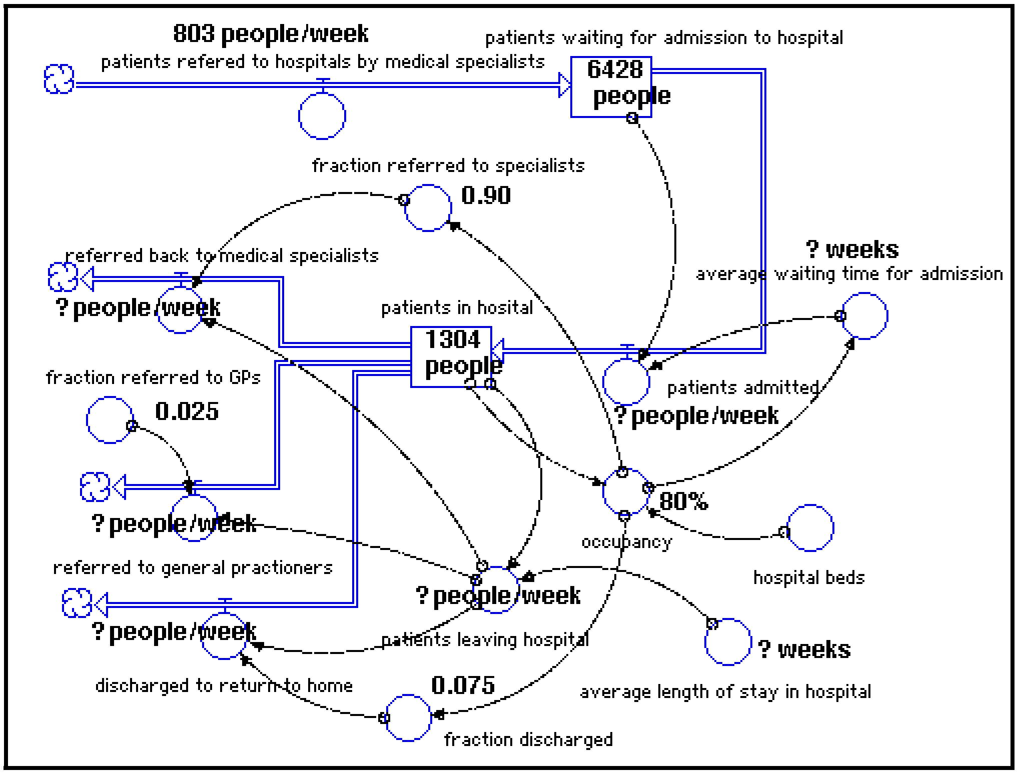

Figure 2. Another equilibrium diagram with missing values

The second stock shows 1,304 people in the hospital. Since the system is in dynamic equilibrium, you can replace two more ?? in the diagram:

What is the value of the "patients leaving hospital"? ______________________
What is the value of the "average length of stay in hospital"? ___________________

Verburgh explains that the around 2.5% of the patients leaving the hospital are referred to GPs (General Practitioners). Depending on the occupancy, around 90% could be referred back to medical specialists and 7.5% could be discharged to return to home. With these assumptions, you can fill in the rest of the equilibrium diagram:

What is the value of the flow "referred to general practitioners"? __________________

What is the value of the flow "referred back to medical specialists"? _____________________

What is the value of the flow "discharged to return to home"? _____________________

3. Three Bottle System in Equilibrium

Exercise 6-9 in the book asks you to describe the equilibrium conditions for the two bottles system shown in Figure 3. Exercise 6-10 asks you to build the model and then simulate it to confirm that the equilibrium conditions make sense.

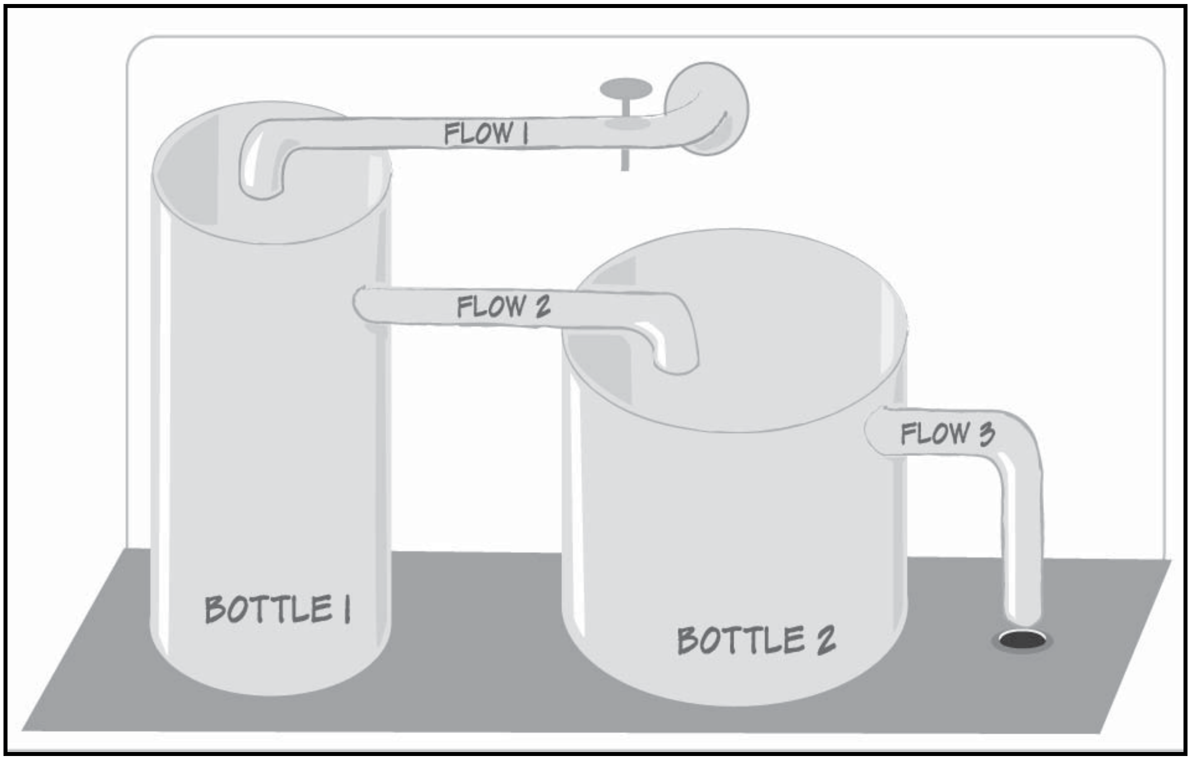

Figure 3. Sketch of the two bottles system (Figure 6.12 in the book)

We now consider a system with three bottles and evaporation. The inflow to the first bottle is the same as before, constant at 5 cc/second. The overflows from bottles 1 and 2 are controlled by the same rules as before. Now imagine that the fluid is highly volatile, so we should account for evaporation from the fluid surfaces. The evaporation rate is 0.1 cm/second.

For this exercise, a third bottle will be used to collect the overflow from the 2^{nd} bottle. (The fluid no longer goes down the drain.) The third bottle is a cone shaped container. The cone is much larger than the other two bottles, and it has a very large capacity. This means we don't need to worry about any overflow from the third bottle.

From this information, you should be able to answer the following questions:

What is the overflow from the 1st bottle when the system reaches equilibrium? ______________

What is the overflow from the 2nd bottle when the system reaches equilibrium? ______________

What is the evaporation from the surface of fluid in the 3rd container? ___________________

What is the surface area of the fluid in the 3rd container? _____________________

4. Build the Three Bottles Model and Verify Your Description of Equilibrium

Expand the Two Bottles Model from exercise 6.10 in the book to account for evaporation from all three surfaces. Introduce a third stock to accumulate the fluid in the third container. Let's assume that the cone is shaped at a 45 degree angle, so the volume of the cone is one-third of the volume of a cylinder with the same surface area and height. This means we can write a geometric equation for the volume:

V = A*H/3 where

V = Volume of fluid in the cone

H = Height of fluid in the cone

A = surface Area of fluid exposed at the top of the cone

and another equation for the height:

H = Cube Root (3*V/3.1346)

where 3.1346 is pi, the Greek letter used to characterize circles. By the way, Stella has a "square root" function in the list of special "built-in" functions, but not a cube root. To take the cube root, you may use the ^ symbol which stands for "raise to the power." For example, X^3 stands for Xcubed and X^(1/3) stands for the cube root of X.

Build this model and simulate it for a sufficiently long period to allow the system to reach equilibrium. Turn in a time graph of the overflows from bottles 1 and 2, the evaporation from the fluid in the cone and the surface area of the fluid in the cone. Does the simulated equilibrium match your answers to the previous exercise?

5. Physiological Example

Turn to the "first model" of body temperature control explained in the physiology exercises on this website. Do the heat flows in and out of the body core produce a dynamic equilibrium?

If not, suppose you were to add a flow of heat into the core from shivering. (Shivering is an involuntary contraction of the muscles to release chemical energy into the core.) What is the value of the shivering heat flow to achieve dynamic equilibrium in the body's core?

6. The Ragwort Plant and Its Seeds

Turn to the exercises on the Ragwort Plant on this website. You will see a complicated model with four stocks and ten flows. When the model reaches equilibrium, there will be 4 leaved plants and 1 flowering adult plant in the test area. Now, suppose we were to double the number of "invading seeds" from 5,000 seeds/yr to 10,000 seeds/yr. What do you think will happen to the number of flowering adults in the test area when the system reaches equilibrium?

(Hint: You might work through the diagram based on a gut instinct that there will be twice as many flowering adults. Change the number of invading seeds to 10,000 seeds/yr and the number of flowering adults to 2 plants. Then work your way through the rest of the equilibrium diagram to check whether your gut instinct is reliable.)

7. Confirm equilibrium values of forest land use:

In chapter 6 of the book, you were asked to estimate the size of the five stocks in Fig. 6.6 for dynamic equilibrium. The total area is 97,067 acres and the mature reserves are 42,667 acres, which leaves 54,400 acres for the other five stocks. Build the model in Fig. 6.6 with the harvest fraction set to 0.0375/yr. Set the initial values of the five stocks to 10,880 acres to guarantee that they add up to 54,400 acres. Simulate the model over a sufficiently long time period for the forest to reach equilibrium. Do the equilibrium values match your estimates?

8. Confirm forest harvest policy results:

The model in the previous exercise can be used to study harvest policies, as you can confirm by reproducing the results in Figures 4 and 5. Ask for an 80 year simulation with a pause interval of 5 years. Simulate the first 20 years with 5% annual harvesting, and you will see over 41,000 acres of mature reserves and a harvest of 2,050 acres/yr. The bar chart shows that mature reserves dominate the land use categories. Now, suppose the land owners expect a big increase in lumber prices in the coming decades, and they want to take advantage of their large reserves. They bring in a huge workforce and harvest at 40% per year. With such large reserves, they are confident that this new policy will deliver large harvests over several decades.

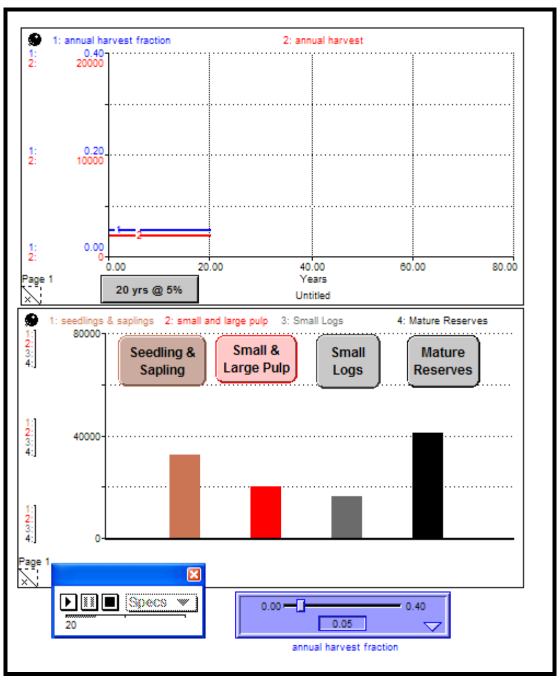

Fig. 4. First 20 years of a policy test.

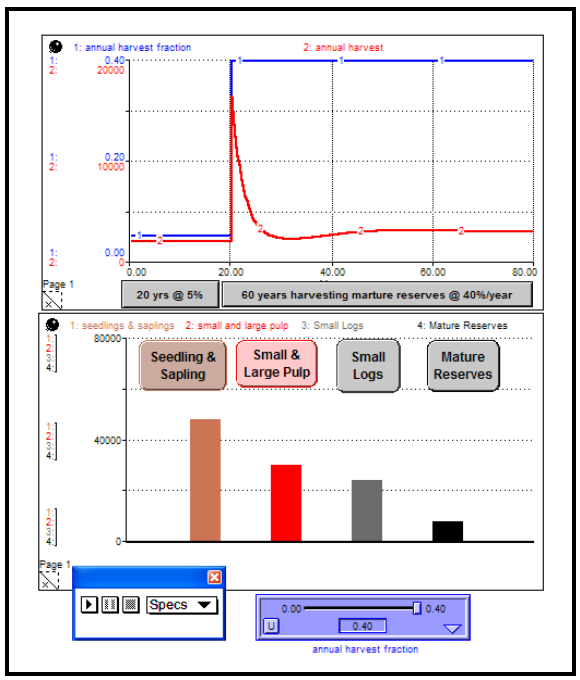

Fig. 5. Remainder of the policy test.

Fig. 5 shows that the land owners would be disappointed. There is a sharp increase in harvest after the 20th year. However, by the 30th year, the annual harvest is about the same as before. (The large work force engaged in the previous few years would now be gone.) Fig. 5 shows that annual harvest would then gradually increase and reach an equilibrium around 3,000 acres/yr by the 50th year. Meanwhile, the land use categories have changed dramatically. The dominant category is now seedlings and saplings while the mature reserves are the smallest bar in the chart. They would occupy only 7,500 acres and provide the land owners with a small cushion (in case lumber prices should increase again.)