

Limnology: Accelerated Eutrophication in a Lake with Nitrogen and Phosphorous Limitations

Accelerated eutrophication of lakes is usually associated with excessive inflows of nutrients such as phosphates, nitrates and ammonium. The flow of nutrients into the lake can allow for rapid increase in algae. The alga is relatively short-lived, and the lake soon exhibits high concentration of dead organic matter which starts to decay. The decay process consumes dissolved oxygen in the lower layer of the lake. Without sufficient dissolved oxygen, animals and plants may then die off in large numbers.

Computer simulation modeling can aid our understanding of the dynamics of accelerated eutrophication, and it can help us test policies to deal with the problem. For example, models may help us anticipate the impact of buffer zones (to reduce nutrients reaching the lake) or the impact of aeration to rebuild the oxygen levels in a lake that is suffering from hypoxic conditions. The starting point for such models is a simulation of the nutrient cycling in the lake.

Purpose of the Exercises

These exercises describe a model of nutrient cycling in a fresh-water lake. The model simulates the biomass and detritus in a lake with both nitrogen and phosphorous in the nutrient pool. These nutrients are required for growth, and growth may be said to be “N limited” or “P limited” depending on the availability of the required nutrients. The model will be used to illustrate how the limitation on biomass growth can shift from N limited to P limited over time.

This transition will be familiar to students of limnology, so the model is a useful starting point for exercises. If you are familiar with limnology, you will immediately recognize that the model ignores a wide variety of factors (i.e., temperature, dissolved oxygen, solar radiation, stratification, nutrients in the sediments and the flushing of the lake by outflows). These and other factors are intentionally ignored so that the exercises begin with a simple model that is easy to understand. The goal is to illustrate how system dynamics can be used to simulate the fundamental concepts of aquatic modeling:

There are only three fundamental concepts employed in the construction of quantitative models of nutrient cycling to aquatic environments as well as used in the qualitative understanding of what occurs therein... The three principles are conservation of mass, kinetics of interactions and stoichiometry of chemical transformations.

(Verhoff 1973, p. 213)

Verhoff considers the **conservation of mass** as the most rudimentary principle. Adherence to this principle will be clearly demonstrated in these exercises by the clarity of stock and flow structure. It will also be demonstrated by stacked graphs of N and P which allow for an easy visual check on the total amounts of N and P in the lake.

The second concept involves the **kinetics of interactions**. Verhoff describes a “stoichiometry limiting approach” in which only one nutrient is limiting at a time. He then describes a “kinetic limiting” approach in which more than one nutrient can be limiting simultaneously. The model in these exercises uses the Michaelis-Menten (M-M) equations to represent the kinetics, and the results are presented in “x-y graphs” to verify that the M-M curves are working properly. The limitations are imposed as if only one nutrient can be limiting at a time. Simulating the lake with simultaneous limitations is left as an exercise.

Verhoff’s third concept is the **stoichiometry** of chemical transformations. These exercises assume that the relative concentrations of C:N:P adhere to the Redfield (1958) molar ratios of 106:16:1. The stocks in the model will be measured in micro moles per liter (umol/L). So, if the stock of carbon (C) in the biomass were 106 umol/L, the model should show nitrogen (N) in the biomass at 16 umol/L and the phosphorous (P) in the biomass at 1 umol/L. The model also adheres to the 106:16:1 molar ratios when simulating biomass growth, biomass death and detritus decay.

The model simulates time in months, with a typical simulation lasting 48 months. This short time interval will show four years with seasonal changes within each year. The four-year interval is sufficient to simulate a transition from N-limited to P-limited conditions when there is a high inflow of N to the lake.

The model is implemented in Vensim with five views: two views for the diagrams and three views for changing inputs and viewing the results. The model is also implemented in Stella, and the BWeb provides a separate document with the Stella version of the images shown in this document. Both the Vensim and Stella models may be downloaded from the BWeb.

Illustrative Results

We simulate the model over 48 months with seasonal changes in the biomass growth rate. The model shows rapid growth in biomass in the growth season (months 5, 6, 7, and 8) followed by rapid growth in the detritus. A 48-month simulation will allow the seasonal pattern to appear four times. It will also allow sufficient time for dramatic growth in biomass if there is sufficient N and P in the nutrient pool.

Fig 1 shows this simulation from the main view of the model. The three graphs are stacked graphs with the top of the graphs showing the total amount of C, N or P in the lake. The first graph shows the carbon in the detritus stacked on top of the carbon in the biomass. Biomass and detritus grow dramatically in the first 24 months of the simulation. This growth is made possible by the growth in nitrogen in the nutrient pool. The N inflow is controlled by the slider at 2 umol/L per month. Since the N inflow is constant for the entire simulation, we would expect total N in the lake to grow in a linear manner. The stacked graph confirms this expectation. There is no P inflow, so we expect the total P in the lake to remain constant, as confirmed in the third graph. Although the total P in the lake is constant, the distribution of the P changes over time. The simulation begins with most of the P in the nutrient pool. By the end of the simulation, most of the P is embedded in the detritus.

The growth of biomass is only possible if there is sufficient N and P in the nutrient pool. Figure 1 shows much less¹ N than P in the nutrient pool when the simulation begins. This would suggest that growth in biomass would be limited by the availability of N. However, by the end of the simulation, there is a lot less P than N in the nutrient pool. The relative shortage of P is responsible for the leveling off of the biomass and detritus in this simulation.

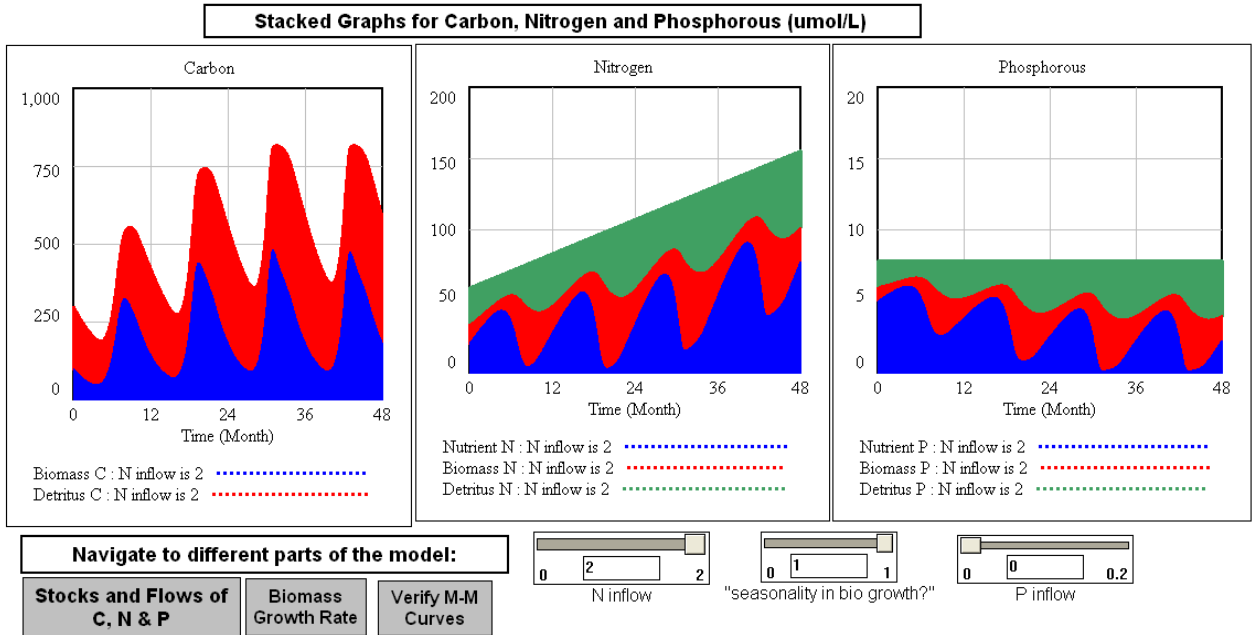


Fig. 1. Illustrative results with N inflow at 2 umol/L per month.

Core Structure

Fig. 2 shows the main stocks and flows in the model. The diagram uses color to distinguish between the carbon (in black), the nitrogen (in red) and the phosphorous (in purple). We use the initials C, N and P throughout the paper. Fig. 2 shows the flow “biomass net C growth” adding to the stock of Biomass C. This stock represents umol/L of C of primary producer biomass (phytoplankton). Phytoplankton takes up carbon as it grows and gives back carbon through respiration. The “net” in the net growth rate means we are simulating growth minus respiration. The biomass C death transfers the C from biomass to detritus, and the decay of detritus sends the C to a sink. (The decay rate is 0.2/month.) The cloud symbols mean we are not keeping track of the C in the nutrient pool.²

¹ The Redfield molar ratios tell us to expect the 16 times more N than P in the biomass and the detritus. Notice that the vertical axis on the N graph is 10 times larger than the vertical axis on the P graph. With these numbers in mind, Figure 1 gives a good indication of the relative availability of N and P in the nutrient pool. The graphs indicates there is relatively little N at the start of the simulation and relatively little P at the end.

² Although carbon can sometimes act as a limiting nutrient, nitrogen and phosphorous are the “more common suspects” in most lakes (Anderson 1973, 124).

Stocks and Flows for Carbon Nitrogen & Phosphorus Return

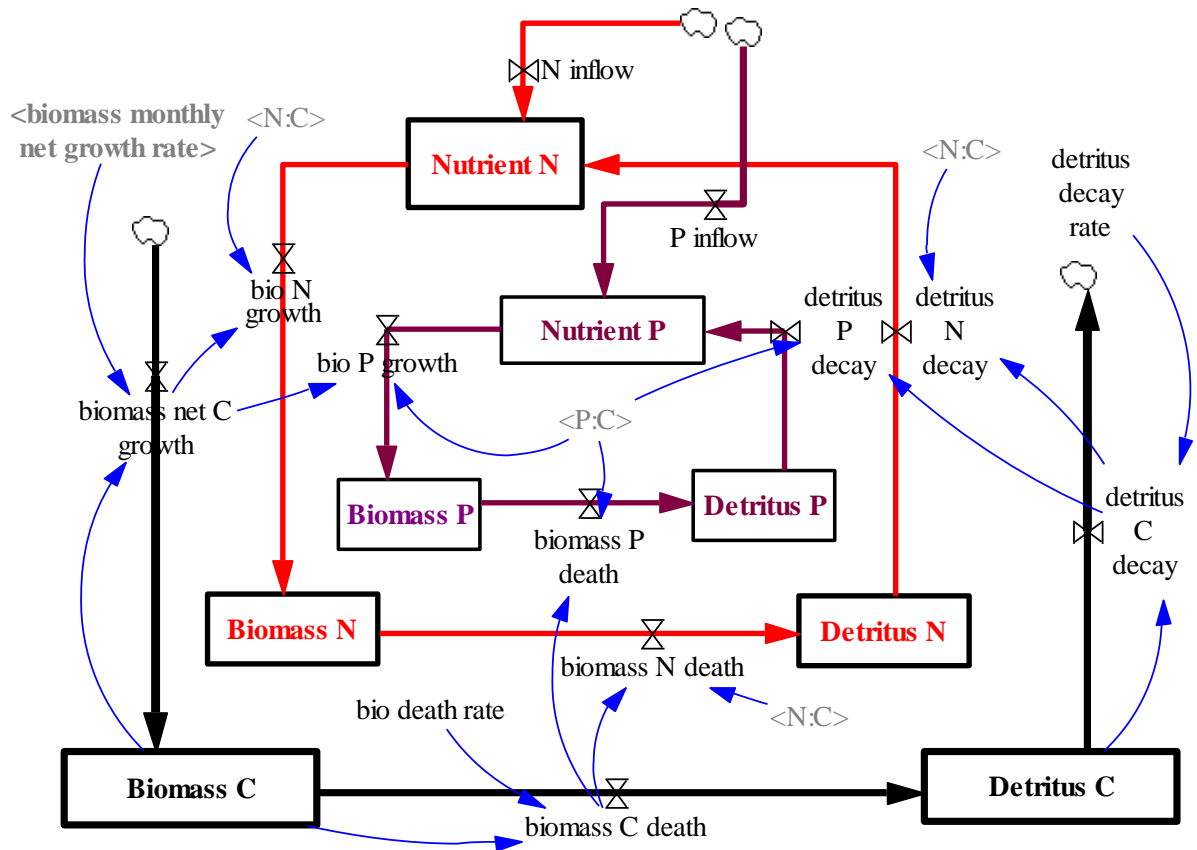


Fig 2. Core structure of the lake model.

The red flows represent the N cycling in the lake. The stocks are measured in $\mu\text{mol/L}$. The N in the nutrient pool represents the inorganic forms of N, typically nitrate (NO_3^-) and ammonium (NH_4^+). N is removed from the nutrient pool through growth of the biomass, and the N flows are proportional to the corresponding flows of carbon. The N:C molar ratio is set at 16/106 based on the Redfield molar ratios.

The purple flows represent the P cycling in the lake, and the stocks are also measured in $\mu\text{mol/L}$. The P in the nutrient pool is phosphate (PO_4^{3-}). P is also removed from the nutrient pool through growth of the biomass, and the P flows are proportional to the corresponding flows of carbon. The P:C molar ratio is 1/106 based on the Redfield molar ratios.

The inflows of N and P are specified by the user with the sliders shown in Fig. 1. (The inflows could represent deposition or fertilizer runoff from agriculture in the watershed.) The inflows act to fertilize the lake and increase production of biomass. The simulation in Fig. 1 assumes that no phosphate enters the lake, but a significant amount of nitrate and ammonium does enter the lake.

Fig 3 shows the variables used to establish the initial conditions.³ The model adopts the following concentrations for the start of the simulation:

- Nutrient P = 5 umol/L: With a molar weight of 32 ug/umol, the initial concentration is 160 ug/L. This is at the “very poor” end of the scale of water quality by the Wisconsin Department of Natural Resources (Wisconsin DNR 2011).
- Nutrient N = 20 umol/L: With a molar weight of 14 ug/umol, the initial concentration is 280 ug/L.
- Biomass C = 100 umol/L: With a molar weight of 12 ug/umol, the initial concentration is 1,200 ug/L or 1.2 mg/L. This initial value was selected to yield repeating conditions from one year to the next in simulations with no inflow of N or P. In other words, 100 umol/L is consistent with the many other parameters and assumptions in the model.
- Detritus C = 200 umol/L: With a molar weight of 12 ug/umol, the initial concentration is 2,400 ug/L or 2.4 mg/L. This value was selected to yield repeating conditions from one year to the next in simulations with no inflow of N or P.

The remaining initial concentrations are governed by the Redfield molar ratios. For example, the initial value of the Biomass N is 100 umol/L of carbon multiplied by 16/106. So we start with 15 umol/L of N embedded in the biomass. Similarly, the initial value of the Biomass P is 100 umol/L of carbon multiplied by 1/106. So we start with 0.94 umol/L of P embedded in the biomass.

The initial amounts of N and P in the Detritus are set in a similar fashion. The simulation begins with twice as much Detritus C as Biomass C. So the initial amounts of N and P in the detritus will be twice as large as in the biomass.

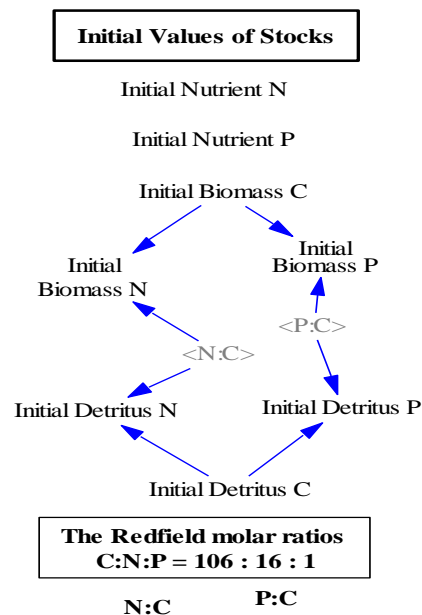


Fig. 3. Setting the initial values.

³ The initial values of almost all stocks in the book are specified by entering a number in the appropriate position in the Vensim stock (level) equation. However, it is sometimes useful to be assign a named variable to the initial value (i.e., so the value can be controlled with a slider.) We add the named variable to the model and then add an arrow from the named variable to the stock variable. Then open the equation for the sock and click on the named variable. When you close the equation for the stock, Vensim will remove the arrow. This is an important visual feature of the model diagrams because only the arrows for the flows enter or leave the stock. This visual feature reminds us that only the flows act on the stocks once the simulation begins.

The Kinetics of Biomass Growth

Figs 1, 2 and 3 show that the model adheres to the principle concepts of conservation of mass and the stoichiometry of chemical transformations. We turn now to the kinetics of interactions, the third of Verhoff's fundamental concepts. The key variable is the "biomass monthly net growth rate," the 'shadow variable' in the upper left corner of Fig. 2. Fig. 4 shows the separate view where the net growth rate is calculated. The variables on the left allow for seasonality. The question-marked variable "seasonality in bio growth?" assumes that 1 means yes and 0 means no. If this binary variable is set to 0, the net growth rate will be 60% per month in every month of the year. But if the binary variable is set to 1, the net growth rate will be shaped by the monthly multiplier. The lookup sets the multiplier at much higher growth in the 5th, 6th, 7th and 8th months of the year.⁴

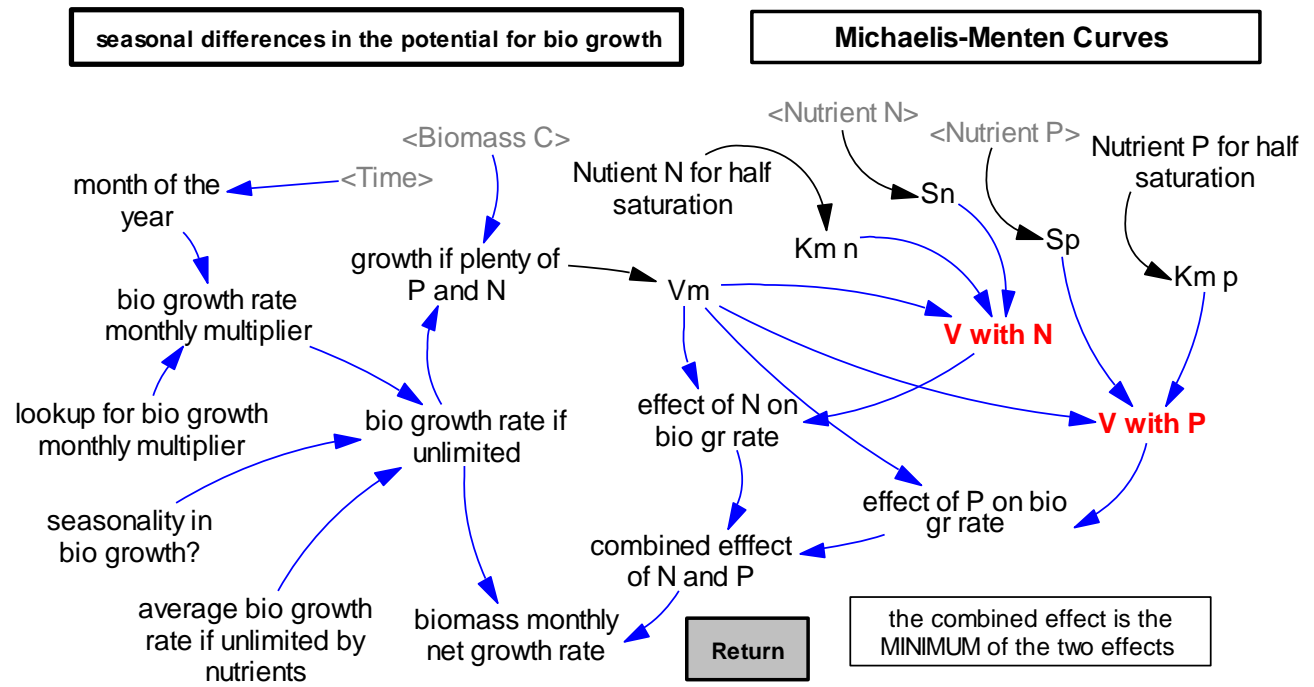


Fig. 4. Seasonality effects and the nutrient limitations on biomass growth.

The variables on the right side of Fig. 4 make use of Michaelis-Menten curves to represent the effect of N and P limitations on the biomass net growth rate. A typical curve is shown in Fig. 5, with the V standing for the biomass growth and V_{max} standing for the maximum possible growth (the growth if there were no limitations from nutrients.) The [S] on the horizontal axis represents the saturation of the nutrient N or P, the $\mu\text{mol/L}$ of N or P in the nutrient pool. The K_m point on the horizontal axis is called the nutrient for "half saturation" because the curve calls for the growth to be half of the maximum value.

⁴Further information on simulating monthly variability is given on page 177 of the book. An entry of 1 means that the growth in that month is the same as the average growth. An entry of 2 means the biomass growth is twice as large. When entering the values, remember to have the total of the 12 entries add to 12. (This is to ensure consistency with the average value over the 12 months in the year.)

The model represents separate limitations from nutrients N and P. The V associated with nitrogen is called V with N. The V associated with phosphorus is called V with P. The equations follow the standard form for the Michaelis-Menten curve

$$V \text{ with N} = \frac{V_m \cdot S_n}{K_m n + S_n}$$

$$V \text{ with P} = \frac{V_m \cdot S_p}{K_m p + S_p}$$

with saturations in $\mu\text{mol/L}$, and the rates in $\mu\text{mol/L per month}$.⁵ The $K_m n$ (the half saturation for N is $16 \mu\text{mol/L}$). The $K_m p$ (the half saturation for P is $1 \mu\text{mol/L}$).

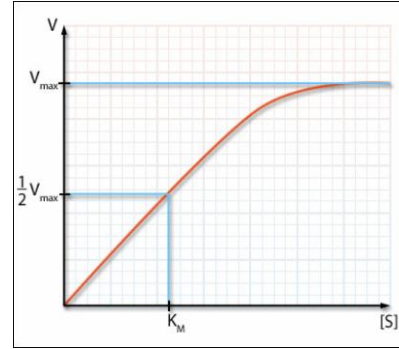


Figure 5. The Michaelis - Menten curve.

The model includes separate graphs to confirm that the Michaelis-Menten kinetics are implemented as intended. Fig. 6 shows the effects of N or P on the net biomass growth rate as a function of the concentrations of N or P in the nutrient pool. The shape of the traced curves matches the general shape shown in Fig. 5. This is a good sign that the model is working correctly. These curves also provide a different way to display model results as many scientists will be familiar with the shape of these curves. Also, take note of the “effect of N” when S_n is at $16 \mu\text{mol/L}$ (the “half saturation” point). The effect is 0.5 as intended. Similarly, the “effect of P” curve is 0.5 when the S_p is at $1 \mu\text{mol/L}$. And finally, notice that Fig. 6 shows a larger range of values for S_n than S_p . The dots in the N graph trace out a greater portion of the curve because N is flowing into the lake during the simulation.

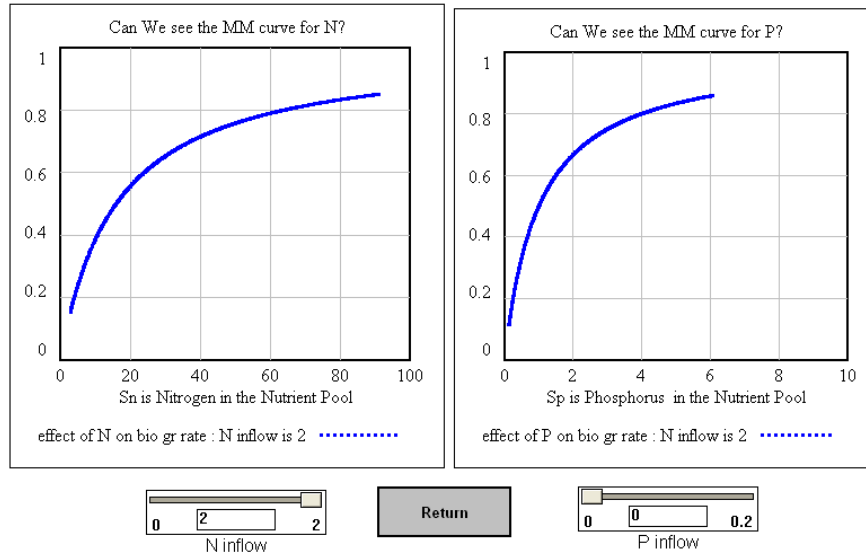


Fig. 6. Graphs to trace the operative portions of the Michaelis-Menten curves.

⁵ Units: The variables K_m and S are measured in $\mu\text{mol/L}$, so the denominator in the M-M equation is also measured in $\mu\text{mol/L}$. The $\mu\text{mol/L}$ in the numerator and the denominator will cancel out, so we know that the V has the same units as the V_m . V_m stands for the maximum “growth rate,” and the M-M descriptions make it clear that the such “rates” would be measured in monthly change in $\mu\text{mol/L}$. Fig. 4 uses the M-M equations to obtain V with P and V with N and then arrives at the corresponding growth rate to be used in the model. The “biomass monthly net growth rate” in Fig 4 is measured as a fractional change in the biomass concentration per month.

The combined effect of the N and P effects is assumed to be the minimum of the two effects.⁶ Fig. 7. Shows the separate effects over the simulation. The blue curve is the effect of N, and the red curve is the effect of P. The first 12-months show dramatic variations in the N effect over the seasons of the year. The effect is 75% at the start and less than 25% during the months 8, 9 and 10. Biomass growth is N limited during the first year. The limitation is especially pronounced in months 8-10. This limitation arises from the rapid growth in months 5-7 which removes N from the nutrient pool.

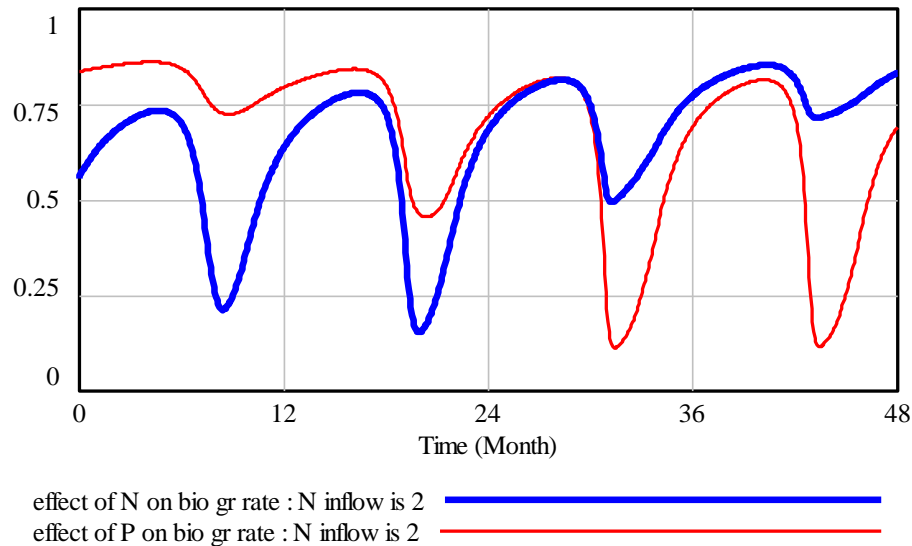


Fig. 7. Nutrient effects on the biomass growth rate.

The second year is a replay of the first year as far as the nitrogen effect is concerned. Fig. 7 shows a greater change in the P effect, but the biomass growth is limited by nitrogen again in the second year. The kinetics change dramatically in the third year, especially when the simulation reaches the 8th, 9th and 10th months. There is no P entering the lake, and the amount of P left in the nutrient pool in months 8-10 of the third year is insufficient for the growth of biomass. Fig. 7 shows that the P limitation is dominant during the third and fourth years of the simulation.

An Illustrative Policy Test

Suppose the growing load of detritus and biomass in Fig 1 is recognized as a problem for the lake and the people who depend on the lake for water supply and for recreation. And suppose the people are convinced that the nitrate and ammonium that is leaking into the lake is responsible for the heavy load of biomass and detritus. Fig. 8 shows a simulation mid-way through the 48 month interval.⁷ Now suppose the people take action to cut the N inflow completely to zero. Would they see biomass and detritus return to the lower levels from two years ago?

⁶ A numerical example will illustrate. Suppose the nutrient concentrations are close to their “half saturation” points, and the separate effects are 0.51 for P and 0.49 for N. Their combined effect would be 0.49 and it is due entirely to the lack of sufficient N in the nutrient pool. In other words, only one nutrient is limiting at any time. Lake dynamics with both nutrients limiting simultaneously is left as an exercise.

⁷ This simulation was created in the “game” mode. The N inflow would be specified as an “auxiliary” and “game” type in the windows to the left of the numbers pad when writing the equation. The “game” mode is available in PLE+ but not with PLE. If you use PLE, you could change the N inflow from 2 to 0 when time exceeds 24 using the IF THEN ELSE function.

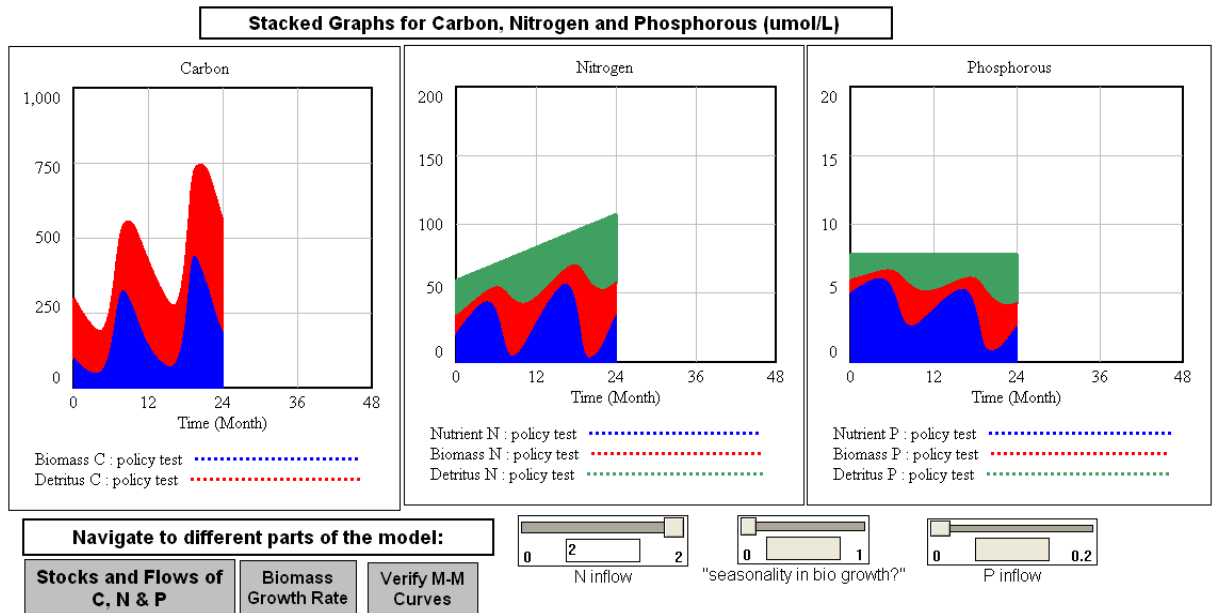


Fig. 8. First half of a policy simulation with the N inflow at 2 umol/L per month.

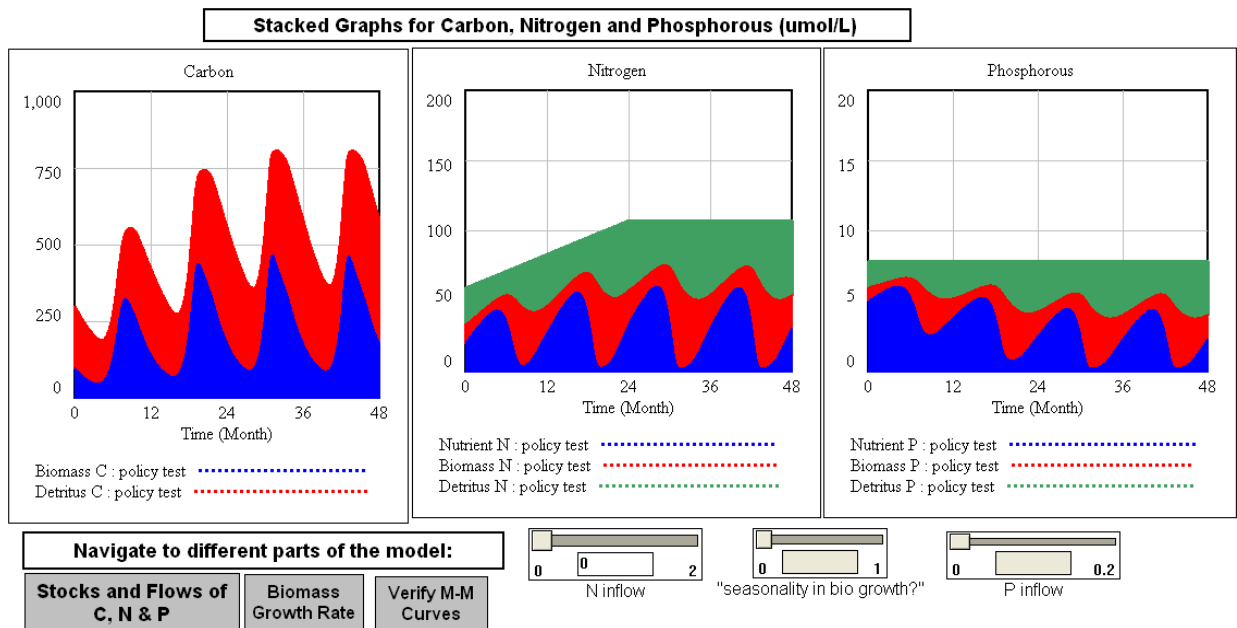


Fig. 9. Rest of the policy simulation with N inflow cut to zero.

Fig. 9 shows that the people would be disappointed. Despite their success in cutting N inflow entirely to zero, the biomass and detritus grow to the same levels shown previously in Fig 1. And they would certainly be disappointed if they expected the carbon and detritus to decline to the levels seen in the first year of the simulation. The simulation shows that the reduction in N inflows has no effect because the lake has already made the transition from N limited to P limited conditions.

Summary and Downloads

The goal of this simple model is to demonstrate how system dynamics may be used to implement the fundamental principles of aquatic modeling. The model represents the conservation of mass, the kinetics of interactions and the stoichiometry of chemical transformations. The illustrative simulation shows the hypothetical lake making a rapid transition from N-limited to P-limited conditions. The model may be used for sensitivity testing to learn the importance of input parameters, as you are challenged to do in some of the exercises below.

The model may also be used for policy testing. The illustrative policy results presented here may come as a surprise to those who hoped that preventing further nitrogen inflows would return the lake to the conditions at the start of the simulation. On the other hand, students of limnology would see the policy simulation as confirmation of their understanding of what would happen in a lake that has made the transition from N limited to P limited conditions.

The model is a starting point for improvements, some of which are explained in the exercises below. The Vensim model is described here, and a Stella version is described in a separate BWeb document. Both models may be downloaded from the BWeb.

Exercise Collections

The exercises in this document are **verification exercises**. They challenge you to build the model and verify that you get the expected results. The best way to learn is to build the model on your own and try to reproduce the results. If you have problems, download the model from the BWeb and compare the BWeb model with your own.

The BWeb includes **causal loop diagramming** exercises to better understand the feedback loops in the model.

The BWeb includes **oxygen exercises** to represent the dissolved oxygen in the epilimnion and the hypolimnion. The oxygen exercises also challenge you to expand the model to simulate the nitrogen that accumulates in the sediments of the lake. These exercises will lead us to a model that could simulate the benefits of aeration to rebuild the oxygen concentration in a eutrophic lake.

Verification Exercises

1. Build, verify, download and compare: Build the model from the description given here and simulate it for 48 months with DT set to 0.125 (1/8th of a month) and the decay rate at 0.2/month. Compare your results with the results in Figs 1, 6, and 7 to verify that your model is working correctly. Then download the BWeb model and compare it with the model that you constructed.
2. Why the small DT? You may be wondering about the DT = 1/8th of a month. Do we really need such a small step size? Try simulations with DT = 1/4th and 1/2th of a month, and pay particular attention to the P limitation effects in the final year. Do you see “ringing” in the simulations with the higher values of DT?
3. Simulation with no nutrient inflows: What do you think will happen to the biomass and detritus if both the N inflow and the P inflow are set to zero? Run this simulation and study the graphs to verify that the 2nd, 3rd and 4th years are essentially a replay of the 1st year.
4. Higher value of the maximum biomass growth rate: This input is set at 60% per month in the illustrative simulation. This is an uncertain input, so we should consider sensitivity tests. Rerun the model with 80% per month and look at the total of C in biomass and detritus at the end of the 48-month simulation. You should see essentially the same totals as in Fig. 1. After verifying the similar results, explain why the higher value of the maximum growth rate does not lead to higher levels of biomass in the lake.
5. Lower value of the maximum biomass growth rate: Rerun the model with 40% per month and let this simulation run for 60 months. Then look at the total C in biomass and detritus in the final year of the new simulation. You should see essentially the same totals as in Fig 1. After verifying the similar results, explain why the lower value of the maximum growth rate does not lead to lower levels of biomass in the lake.
6. Simulate over longer time intervals with lower N inflows: Fig. 1 shows rapid growth in biomass which is caused by the large N inflow. The 2 umol/L monthly inflow allows the model to complete the transition to P-limited within 48 months. For this exercise, you will cut the N inflow by tenfold and expand the length of the simulation by tenfold. Change the *model settings* to simulate for 480 months and set the monthly N inflow at 0.2 umol/L. You know from the 2nd exercise that DT must be 1/4th of a month, so the model will require 1,920 steps. Before simulating the model, write down your expectation on the likely behavior. Will the biomass stop growing at some point in the simulation? If so, when? Will the amount of biomass and detritus in the lake be similar to the amounts at the end of Fig. 1? Now simulate the model. Do the simulation results match your expectations?
7. Verify the impact of seasonality: Fig. 1 shows major surges in biomass growth in months 5-6-7 of each year. This growth removes N and P from the nutrient pools, and the growth in subsequent months is much lower than you might have expected. What do you think will happen to the biomass and detritus if we remove the seasonal effects? Perhaps the biomass will grow to higher levels if we no longer have growth surges that remove C and P from the nutrient pool? Write down your expectations (before reading further). Then rerun the illustrative simulation from Fig. 1 with the “seasonality” slider changed to 0. You should see the results in Figs 10 and 11.

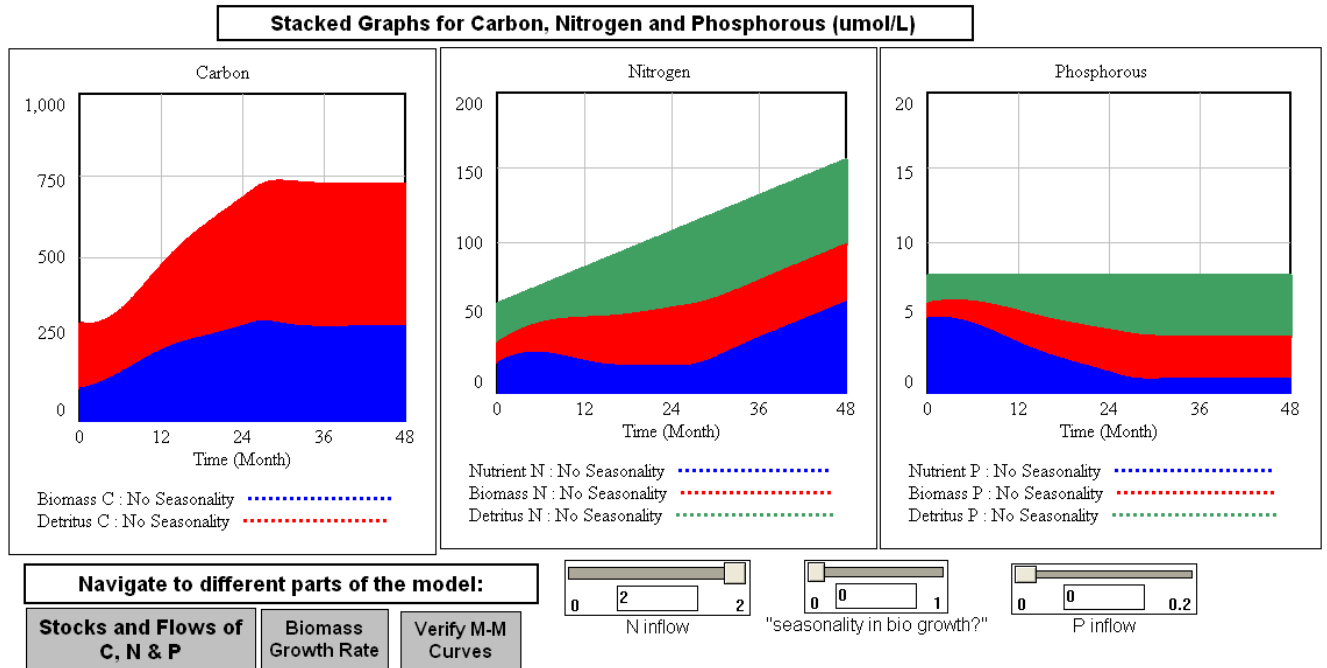


Fig 10. Simulation without seasonal changes (compare to Fig. 1).

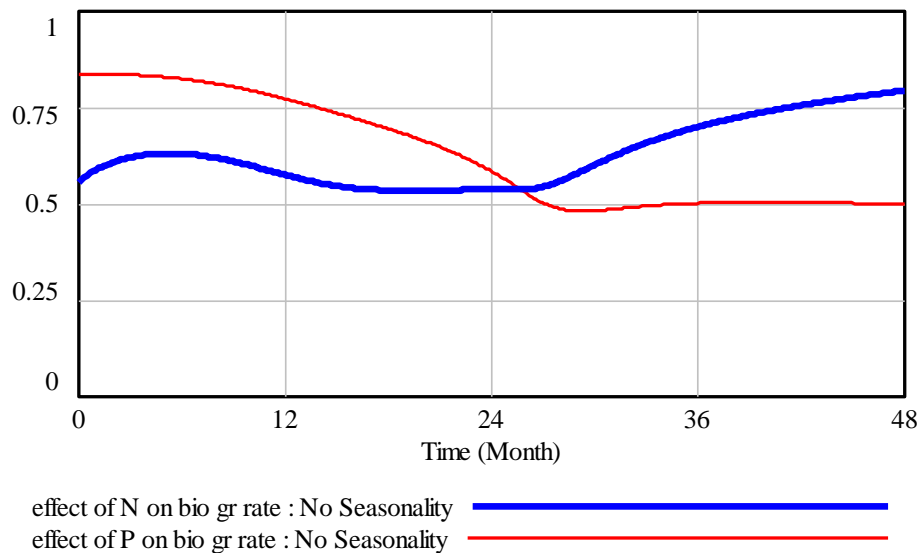


Fig. 11. Nutrient effects in a simulation without seasonal changes (compare to Fig. 7).

8. P Inflow rather than N inflow: What do you think will happen to biomass and detritus if we prevent the N inflow but there is P inflow to the lake. (Think of a policy to eliminate nitrate and ammonium from getting into the lake. But, for some reason, we end up with phosphates entering the lake instead.) Will we see essentially the same biomass and detritus results as shown in Fig 1? Test your thinking by running the model with N inflow at zero and the P inflow at 0.2 umol/L per month.

9. Sensitivity test on the combined effect of N and P limitations: The combined effect of the N and P limitations is represented as the minimum of the two effects. (This is the approach taken in the Pamolare (2010) model.) The minimum function would cause only one of the nutrients to be limiting at a time. But Verhoff describes a “kinetic limiting” approach in which more than one nutrient can be limiting simultaneously. This approach could be simulated in the model by setting the combined effect as the product of the two effects.⁸ Change the equation for the combined effect of N and P and simulate with the assumptions used in Fig. 1. Do you see noticeable changes compared with Fig. 1? Then simulate the model to compare with the “policy test” results in Fig. 9. Do you see noticeable changes?

Further Reading

System dynamics modeling of nutrient flows in aquatic systems is described by the following authors:

- Jay Anderson (1973) describes a Dynamo model of cultural eutrophication in a hypothetical lake with carbon as the limiting nutrient. The model follows system dynamics principles, and the conversion from Dynamo to Stella is relatively clear (see pages 332-333 of the book). However, some of the equations are difficult to understand. The problem seems to be the dual role of carbon – it represents both the mass of the growing biomass and the nutrient that would limit the growth of the biomass. The dual roles of carbon lead to “second order growth equations” which are difficult to explain. A Stella version of Anderson’s model was previously used for extra exercises on the website for the 1st edition of *Modeling the Environment*. But the model has been removed from the website since carbon is seldom the limiting nutrient and because the model exhibited ringing behavior when the biomass reached high levels.
- Michael Deaton (1999) has published a text on modeling of environmental systems in Stella. Chapter 5 describes a model to simulate dissolved oxygen and biochemical oxygen demand.
- Steve Arquitt (2004) describes a Stella model of toxic blue-green algae blooms in Morton Bay, Queensland, Australia. The model is explained in a special issue of the *System Dynamics Review* and may be downloaded from the issue’s website: <http://www.wsu.edu/~forda/SIOpen.html>
- Erica Gaddis (2007) describes a Stella model of diffuse sources of nitrogen, developed with stakeholders in a residential watershed in Calvert County, Maryland.

⁸ The previous numerical example will illustrate the combined effect. Suppose the two nutrient concentrations are close to their “half saturation” points -- the separate effects might be 0.51 for P and 0.49 for N. In this case, their combined effect would be 0.49, and it is due entirely to the lack of sufficient N in the nutrient pool. We would say that only one nutrient is limiting at any point in the simulation. However, for this exercise the combined effect would be the product of 0.49 and 0.51 which is 0.25. The biomass growth rate would be only half as large as in the BWeb model, and one would say that the nutrients are acting simultaneously to slow the growth in biomass.

- Mark Rivers (2009) describes a Stella model of phosphorous movement in the Peel-Harvey catchment in southwestern Australia. The model includes best management practice interventions throughout the catchment along with interface controls to govern the scale and effectiveness of the interventions.
- The United Nations Environmental Program describes Pamolare (2010) -- a structurally dynamic model for shallow lakes. The equations and parameter values are explained in detail (as the UNEP wishes to encourage its use). The model does not use ‘stock and flow’ software like Stella or Vensim. But the stocks and flows are documented in a “conceptual diagram” with numbers assigned to each line.. The numbers direct your attention to the equations for each “process” in the model. The main focus of the UNEP model is calibration for prediction, a process that requires eleven steps and which may “last a few weeks to be made properly.”

References

Anderson 1973

J. M. Anderson, The eutrophication of lakes, in *Toward global equilibrium: collected papers*, D.L. Meadows and D. H. Meadows, editors, Pegasus Communications, Waltham, MA.

Arquitt 2004

Steve Arquitt, A scoping and consensus building model of a toxic blue-green algae bloom, *System Dynamics Review*, 20 (2), Summer 2004, 179-198.

Beutel 1999

Marc Beutel and Alex Horne, A review of the effects of hypolimnetic oxygenation on lake and reservoir water quality, *Lake and Reservoir Management*, 15 (4): 285-297

Deaton 1999

Michael Deaton and James Winebrake, *Dynamic modeling of environmental systems*, Springer, 1999.

Gaddis 2007

Erica Gaddis, Helena Vladich and Alexey Voinov, Participatory modeling and the dilemma of diffuse nitrogen management in a residential watershed, *Environmental Modeling and Software*, 22: 619-629.

Pamolare 2010

Pamolare II: A Structurally Dynamic Model for Shallow Lakes, [IETC-UNEP Software](#).

Redfield 1958

Alfred Redfield, The Biological Control of Chemical Factors in the Environment, *American Scientist*, September 1958.

Rivers 2009

Mark Rivers, David Weaver, Keith Smeltem and Peter Davies, Modeling aggregate catchment-scale phosphorus transport over 200 years in the Peel Harvey catchment, Western Australia, [EGU Poster](#).

Mark has also posted a Stella model (with a Forio front-end interface) to the web:

<http://forio.com/simulate/manager/mrivers/peel-harvey/run.html>

Verhoff 1973

F. H. Verhoff, A note on fundamental modeling principles and algal limiting nutrients, in *Modeling the eutrophication process*, E. Middlebrooks, D. Falkenborg and T. Maloney, editors, Ann Arbor Science Publishers, Inc.

Wikipedia 2010 page on Michaelis-Menten Kinetics

http://en.wikipedia.org/wiki/Michaelis%E2%80%93Menten_kinetics

Wisconsin Department of Natural Resources 2011

Phosphorus – Understanding lake Data, website viewed in March, 2011 at

<http://dnr.wi.gov/lakes/publications/under/phosphor.htm>