BWeb Notes for
Chapter 3: Stocks and Flows

(There are 3 BWeb notes in chapter 3.)

1st Note, page 32, under Fig. 3.2:

Demographers often show charts of populations in 5-year increments to age 80+ years. A popular form is a bar chart with the number of males on one side and females on the other (BWeb).

Figure 1 shows an example for the population of India in 1991. The bars represent the number of people in each category. For example, the lowest bars show around 60 million males and around 58 million females in the 0-4 age group. These bars form the base of the "pyramid diagram." The numbers become somewhat smaller as we move to each successive age group until we reach the number of people aged 75-79. The final bars combine all people at 80+ years of age, and this aggregation breaks the general pattern of the pyramid. But the pyramid shape would continue if you there were additional bars for ages 80-84, followed by 85-89, followed by 90-94, etc. The shape in Fig. 1 is characteristic of rapidly growing populations.

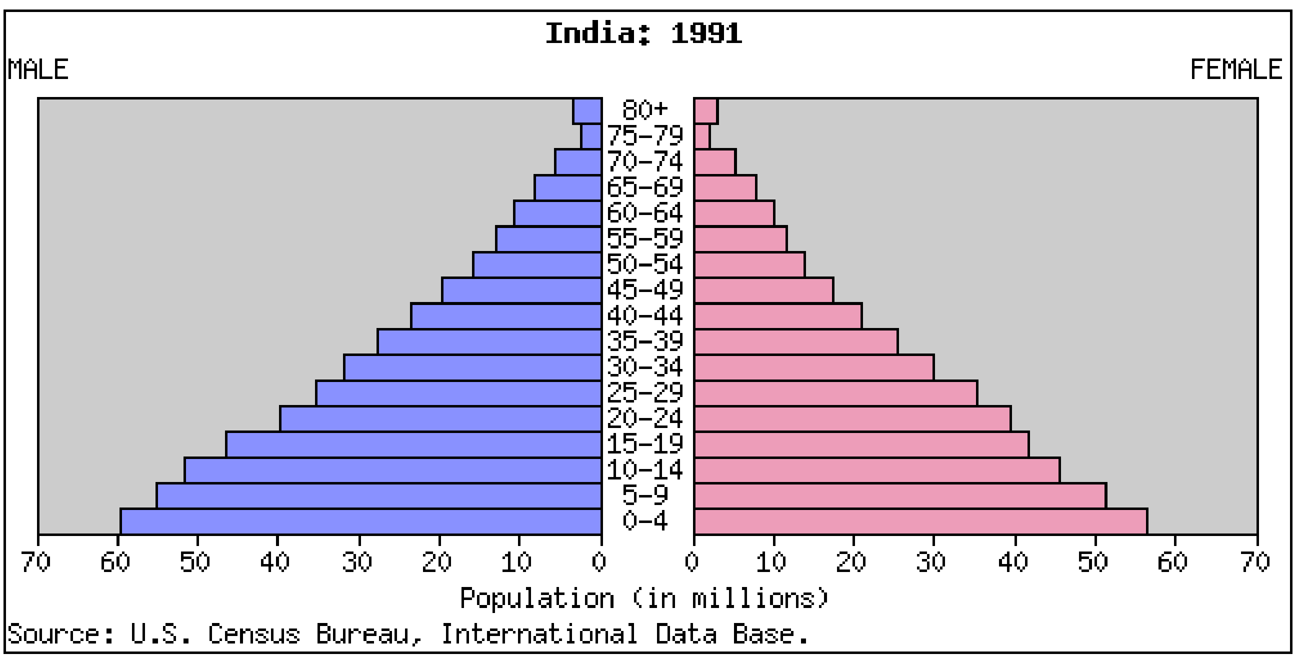

Figure 1. Example of a population pyramid chart for a rapidly growing population.

Pyramid diagrams are an effective way to summarize the current population in a compact form for demographic interpretation. System dynamics models can be used to create such diagrams if the population model is disaggregated to include separate age and sex categories. The population results may be exported to a spreadsheet (or other software) for storage and display.

Pyramid diagrams may also be created within Vensim (DSS version) with user defined graphs. You can learn more from experimenting with the Vensim model POPPYR.MDL, as depicted in Fig. 2. It shows males and females in 14 age categories. You may vary the average lifetime slider, and Synthesim will respond with an immediate change in the population pyramid graph. (This example shows results for an average lifetime of 60 years.)

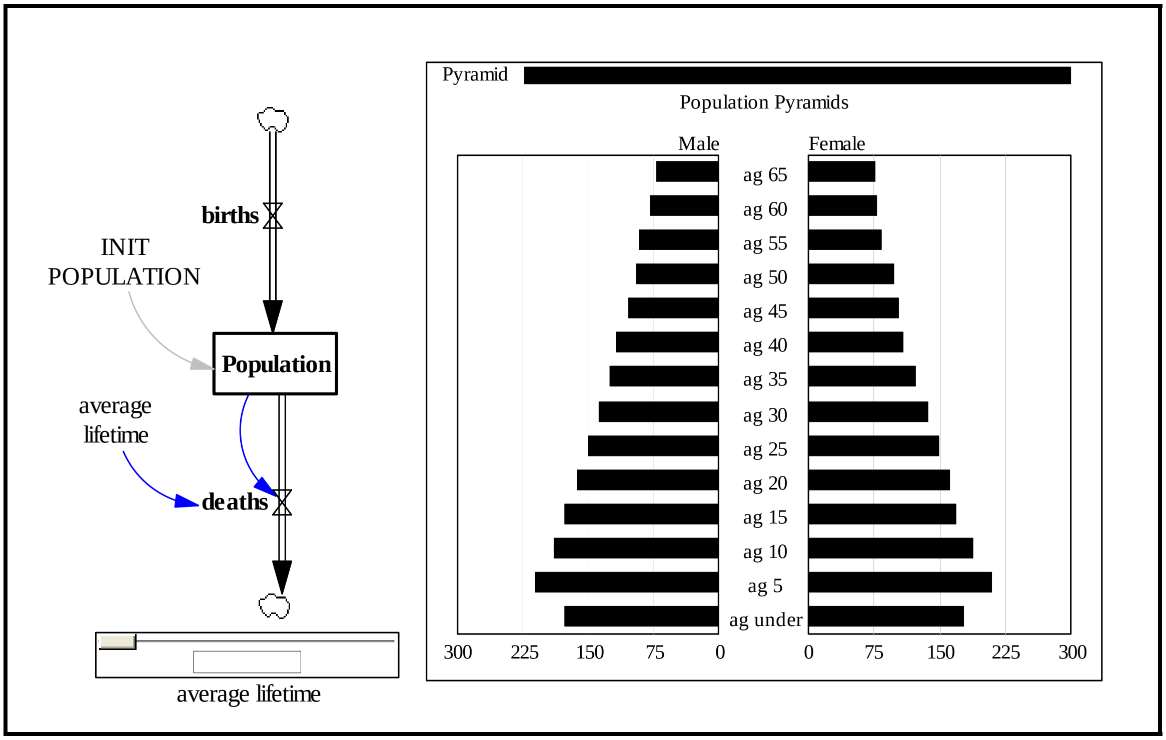

Fig. 2. Vensim model to experiment with the effect on average lifetime on the shape of the pyramid graph.

<u>2nd Note, page 32, bottom of page, referring to Fig. 3.4:</u>

Other students will list the tufa towers, which are prominent in the foreground. The towers are formed by the flow of fresh water from springs, as explained on the BWeb.

You can find the BWeb explanation of the Mono Lake tufa towers at the "Case Information" section of the book's website. The Mono Lake case info is at http://www.wsu.edu/~forda/mb.html

<u>3rd Note in Table 3.1:</u>

The analytical solution to this equation can be found by trial and error (BWeb).

Let's begin by defining some terms:

- P(t) stand for the population as a function of time.
- P(0) = 10, the initial value of the population.
- dP/dt, the derivative of P(t) with respect to t

Then, for any position along the curve, the slope will be equal to the growth, and the growth is equal to the growth rate multiplied by the size of the population. If we let r stand for the growth rate, we can write the differential equation:

dP/dt = rP

This is a first-order differential equation. It also happens to be a linear, differential equation which means we might be able to find a solution. Solving differential equations involves a mix of trial and error, knowledge of some standard solutions, and a willingness to guess the form of the solution based on our intuition about the dynamic behavior. So let's start guessing:

<u>1st attempt</u>
Guess that the solution is P(t) = 10 + t. This guess gives P(0) = 10 as desired and P(t) will grow with time. Now differentiate P(t) with respect to time to get:
dP/dt = 1.
But we were hoping to see dP/dt = rP, so we need to guess again.

<u>2nd attempt</u>
Guess that the solution is P(t) = 10 + rt. This gives P(0) = 10 as desired. Let's differentiate P(t) with respect to time to get:
dP/dt = r
But we were hoping to see dP/dt = rP, so we need to guess again.

<u>3rd attempt</u>
Perhaps the solution is $P(t) = 10 + rt + rt^2$. This guess gives P(0) = 10 as desired. If we differentiate P(t), we obtain:
dP/dt = r +2rt
We still don't have dP/dt = rP.

This line of guess work doesn't seem to be getting anywhere, so it would be useful to ask ourselves about the shape of the population curve over time. We expect to see exponential growth, so our guesswork is likely to be more productive if we work the exponential function into the proposed solutions.

<u>4th attempt</u>
Let's guess that the solution is $P(t) = 10 + e^t$. This guess gives P(0) = 11, which is wrong from the start.

<u>5th attempt</u>
Suppose we guess that the solution is $P(t) = 10e^t$. This guess gives P(0) = 10, and taking the derivative gives
$dP/dt = 10e^t$.
In other words, dP/dt = P. This is encouraging, but we want dP/dt = rP.

6th attempt

Let's guess that the solution is $P(t) = 10e^{rt}$ .This gives P(t) = 10, and taking the derivative gives

$$dP/dt = 10re^{rt}$$

which is the same as $dP/dt = rP$.

It has required 6 attempts, but we have confirmed that

$$P(t) = 10e^{rt}$$

is the solution to the differential equation.

Now, if you've studied differential equations, you might have found this solution with only one or two attempts. But the preceding example is meant to illustrate what happens when searching for the solutions to more complicated differential equations. There are no hard and fast rules that will guarantee that you will find the solution within a few attempts.

This example illustrates another important point -- the key to good guesswork is your own intuition. In this case, we needed to know that populations are likely to grow in exponential fashion. This situation is not unique. As a general rule, we need to have a preconceived image of the answer before the search for a solution is going to converge to the answer. This requirement makes finding analytical solutions similar system dynamics modeling. Specifically, system dynamics modeling will not be productive unless we identify the reference mode (a target pattern) early in the modeling process.