Physics Exercises: Warming of Water in a Glass

Webster's defines physics as a science that deals with matter and energy and their interactions in fields like mechanics, acoustics, optics, heat, electricity, and magnetism. The power of physics lies in describing a great variety of phenomena by a limited set of fundamental laws and principles. Schecker (1996) observes that students sometimes fail to distinguish between a fundamental law and a formula derived in class. For example, all physics students learn the fundamental law about the acceleration of an object: Force = Mass times Acceleration.[1] Schecker cites an example of a student who interprets a fundamental law like:

F = m*A

as *just another equation* of the same quality as

S(t) = 1/2 g t^2

(which is the equation for the free fall of bodies). Schecker feels that system dynamics modeling could help improve how we learn physics by shifting our attention away from memorizing formulas. By focusing on stocks and flows, system dynamics may direct our attention to the key concept of accumulation. I believe his argument make sense, but only in systems where the stocks and flows are easily visualized. These exercises provide an example. We use stocks and flows to keep track of the heat flows in a glass of water.

Glass of Water

The glass of water in Figure 1 is exposed to a constant air temperature of 20 ºC. There are 1,000 cubic centimeters (cc) of water in the glass, and the water temperature is 19 ºC.

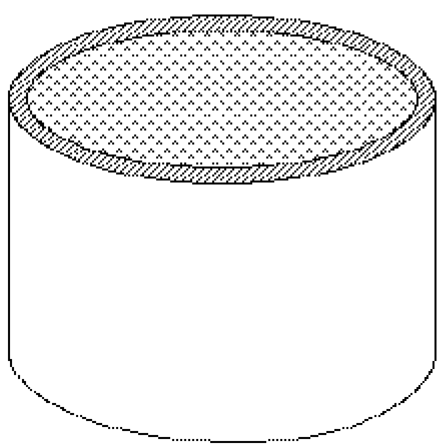

Figure 1. Glass of water

Suppose we know that the air temperature remains constant at 20 ºC. What would you expect the water temperature to be if we come back in a couple of hours? You might answer 20 ºC, the same as the air temperature. Or you might think that the water temperature may not yet be at 20 ºC. But if we wait long enough, it will reach 20 ºC. Now consider a longer term question: suppose we come back in 6 months to measure the volume of water in the glass. Will there still be 1,000 cc in the glass?

You probably realize that evaporation will gradually remove water from the glass, so there will be less than 1,000 cc after 6 months. And you are probably aware that heat is used to evaporate water. If the heat comes from the internal energy stored in the water, we might wonder if the water temperature will decline over time? These exercises develop and test a system dynamics model to simulate whether the water temperature will rise or fall over time.

Heat Flows and Assumptions

The water at the top of the glass has a radius of 5.64 centimeters (cm), a circumference of 35.4 cm and an exposed surface area of 100 square centimeters. The glass is a perfect cylinder, so the exposed surface area will remain constant at 100 square centimeters as the water evaporates. Initially, the water stands 10 cm high. Water density is 1 gram/cc, so the initial mass is 1,000 grams. The glass is 0.5 cm thick, and it sits on a well insulated table in a room with a constant air temperature of 20 ºC and a relatively low humidity. Evaporation takes place at the rate of 2 feet/year. The latent heat of evaporation (the heat needed to evaporate the water) is 585 calories per gram. If we are to study changes in the energy stored in the water, we might consider simulating four heat flows:

- Evaporation (top surface): The heat loss due to evaporation depends on the latent heat of evaporation and the rate at which water evaporates. Let’s include this flow in the model.

- Conduction (side wall): Heat may flow into the water from conduction across the side surface where the water touches the glass. Initially, this surface area is 354 square centimeters. But this surface area will shrink over time as evaporation removes water from the glass. Let’s include this flow in the model.

- Conduction (bottom surface): Heat may flow out of the water through conduction through the bottom of the glass. Let's ignore this flow since the table is well insulated.

- Convection (top surface): Heat may flow into the water by convective forces through the surface area exposed to the air. Let's ignore the convective flow because it would probably amount to only about 1% of the conductive flows across the side surface of the glass.

A Model to Simulate Heat Flows

You know that it's best to "start with the stocks" when building a flow diagram for a new model. And you know that that the stocks represent the storage in the system. In this example, we need two stocks:

- one stock to keep track of the volume of water stored in the glass, and
- a second stock to represent the internal energy stored in the water.

Time will be measured in seconds; volume will be measured in cc; and any flows affecting the volume of water will be measured in cc/second. The internal energy content will be measured in calories, and any flows affecting the energy content will be measured in calories/second. We know that water will gradually leave the glass through evaporation, so we need one flow to account for the reduction in volume as the water evaporates over time. The internal energy content will be controlled by two flows:

- Energy content will be reduced as the water is evaporated. This heat flow depends on the rate of evaporation and the latent heat of evaporation.

- Energy content will be increased if heat flows across the side wall of the glass. This heat flow depends on the temperature difference across the side wall, the thickness of the glass and the conductivity of the glass.

We now have two stocks and three flows as shown in Figure 2. The model is completed by using converters to explain each of the flows.

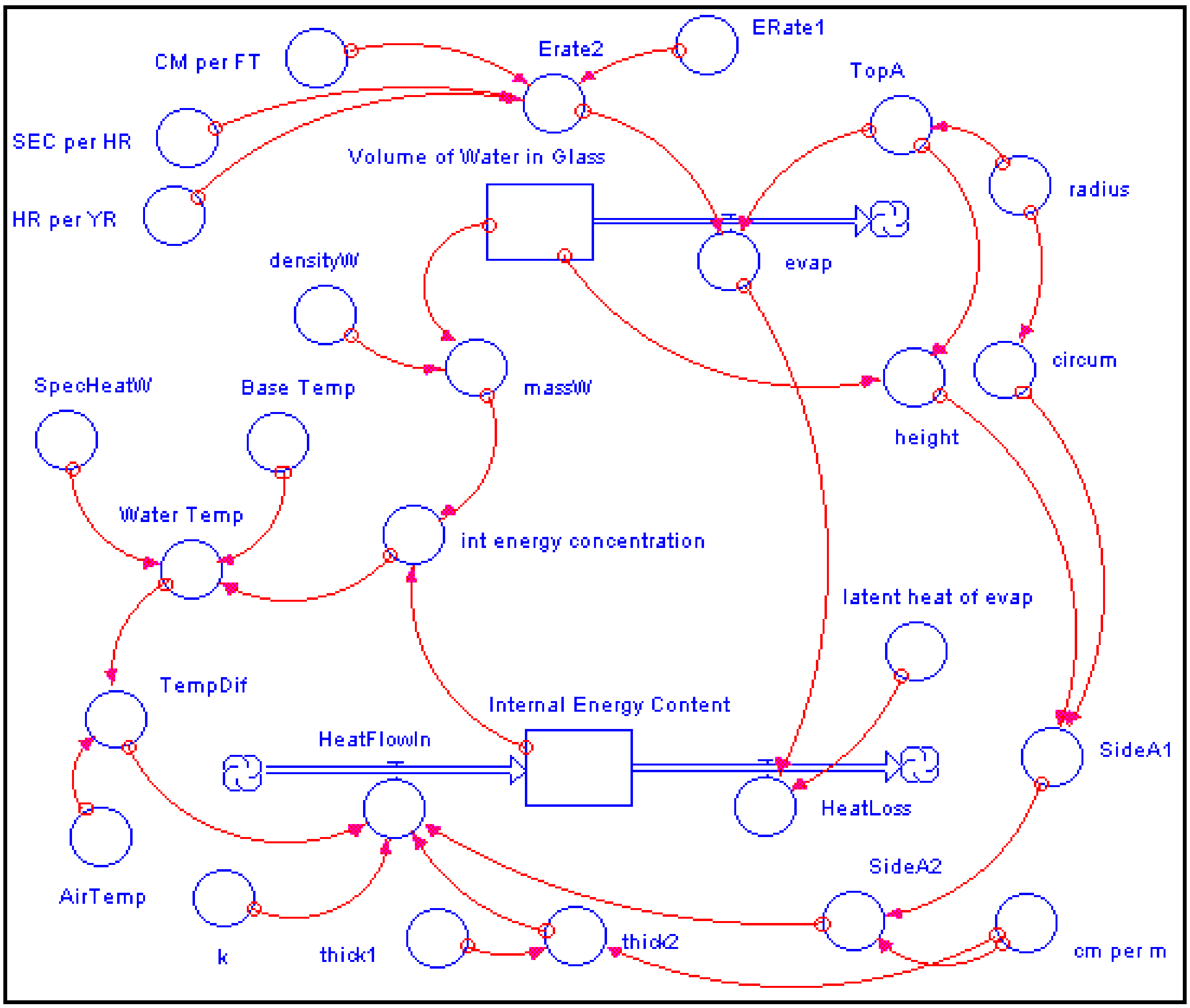

Figure 2. A model to simulate the temperature of water in the glass.

Most of the variables in Figure 2 are converters. You will probably notice that many of the variable names are shorter than names in any of the other models in the book or on the website. I have used short names for the physics exercise because shorter names will make it easier for you to check the model equations against equations in your introductory physics text.

Several of the converters in Figure 2 are inputs (like the air temperature or the density of water). Several are conversion factors. For example, three converters are used to convert Erate 1 (in feet/year) to Erate 2 (in cm/sec).

Most of the units are relatively easy to identify from your introductory course in physics. But the thermal conductivity of glass requires some extra consideration. The value of "k" is 0.2 calories per second-degree C per square meter of glass. This means that the flow across 1 square meter of glass (with a thickness of 1 meter) would be 0.2 calories per second for each degree of temperature gradient across the surface.

Table 1 shows the equations for the stocks and flows. Table 2 shows the equations for the rest of the model.

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Internal_Energy_Content(t) = Internal_Energy_Content(t - dt) + (HeatFlowIn - HeatLoss) * dt
INIT Internal_Energy_Content = 9000
DOCUMENT: internal energy content in calories
INFLOWS:
HeatFlowIn = k*TempDif*SideA2/thick2
DOCUMENT: The heat flow in across the side surface in calories per sec
OUTFLOWS:
HeatLoss = latent_heat_of_evap*evap
DOCUMENT: Heat Loss due to evaporation in calories/second
Volume_of_Water_in_Glass(t) = Volume_of_Water_in_Glass(t - dt) + (- evap) * dt
INIT Volume_of_Water_in_Glass = 1000
DOCUMENT: the volume of water remaining in the glass (in cubic centimeters)
OUTFLOWS:
evap = Erate2*TopA
DOCUMENT: evaporation is measured in cubic centimeters per second
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Table 1. Equations for the stocks and flows.

AirTemp = 20
DOCUMENT: the ambient air temperature in ºC C
Base_Temp = 10
DOCUMENT: an arbitrary base temperature used to establish the temperature of the water
circum = 2*PI*radius
DOCUMENT: circumference measured around the top surface of water (cm)
CM_per_FT = 30.5
DOCUMENT: conversion factor - centimeters in a foot
cm_per_m = 100
DOCUMENT: conversion factor -- centimeters in a meter
densityW = 1
DOCUMENT: in grams per cubic centimeter
ERate1 = 2
DOCUMENT: evaporation rate in feet/year
Erate2 = ERate1*CM_per_FT/(HR_per_YR*SEC_per_HR)
DOCUMENT: evaporation rate in cm per second
height = Volume_of_Water_in_Glass/TopA
DOCUMENT: height of the water in the glass (cm)
HR_per_YR = 8760
DOCUMENT: conversion factor - hours in a year
int_energy_concentration = Internal_Energy_Content/massW
DOCUMENT: calories of energy per gram of water; assumes an even distribution of energy
k = .2
DOCUMENT: k is the conductivity of glass. It has complicated units
latent_heat_of_evap = 585
DOCUMENT: 585 calories are needed to evaporate 1 gram of water
massW = densityW*Volume_of_Water_in_Glass
DOCUMENT: mass of the water in grams
radius = 5.64
DOCUMENT: in cm
SEC_per_HR = 3600
DOCUMENT: conversion factor - seconds in an hour
SideA1 = circum*height
DOCUMENT: area of water along the sides of the glass (square centimeters)
SideA2 = SideA1/(cm_per_m*cm_per_m)
DOCUMENT: side surface area measured in square meters
SpecHeatW = 1.0
DOCUMENT: 1 calorie is needed to raise the temperature of 1 gram of water by 1 degree C.
This is called the specific heat of water.
TempDif = AirTemp-Water_Temp
DOCUMENT: the difference between the air temperature and the water in ºC C
thick1 = .5
DOCUMENT: thickness of the glass in cm
thick2 = thick1/cm_per_m
DOCUMENT: glass thickness in meters
TopA = PI*radius^2
DOCUMENT: area of the top surface of water in square centimeters
Water_Temp = Base_Temp+(int_energy_concentration/SpecHeatW)
DOCUMENT: water temperature in ºC C

Table2. Equations for the rest of the model.

Testing the Model

The simulation begins with 1,000 cc of water and an internal energy content of 9,000 calories. The internal energy content is defined relative to a base temperature of 10 ºC Celsius. The water temperature depends on the Internal Energy Concentration which is measured in calories per gram. At the start of the simulation, there are 9,000 calories evenly distributed over 1,000 grams, so the concentration is 9 calories per gram. The water temperature equation assumes that zero concentration corresponds to 10 ºC. The specific heat of water is 1 calorie per gram per degree. In other words, 1 calorie of heat is required to increase the temperature of a gram of water by 1 degree. Working from a base of 10 ºC, the energy concentration of 9 calories/gram means that the initial water temperature is 19 ºC.

The heat flow into the glass depends on TempDif, the difference between the air temperature and the water temperature which is 1 degree at the start of the simulation. The initial heat flow is around 1.4 calories/sec. But the heat loss due to evaporation is much smaller (only about 0.1 calories/sec). With more heat flowing in than flowing out, you would expect the internal energy content and the water temperature to increase. Let's run the model over thousands of seconds to learn whether the water temperature will eventually reach 20 ºC. Figure 3 shows the results.

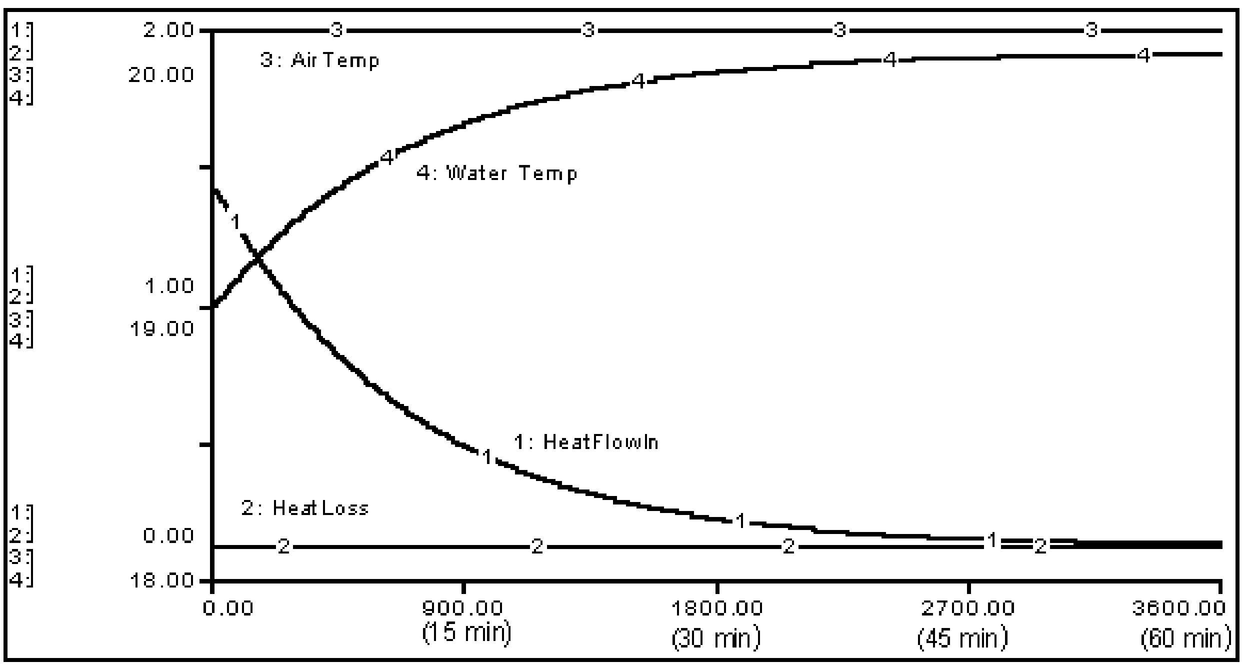

Figure 3. Water temperature over a one-hour simulation.

Remember that time is measured in seconds, so the simulation runs for 3,600 seconds. The comments in ( ) remind us of how many minutes have passed. The air temperature is constant at 20 ºC, and the water temperature increases over time. The heat flows are scaled from 0 to 2 calories/second. At the beginning of the simulation, the heat flowing in is around 1.4 calories/second while the heat loss due to evaporation is only around 0.1 calories/second. The net inflow is over 1 calorie/second. If these flows were to persist for 1,000 seconds, we would expect over 1,000 calories to be added to the energy content of the water--more than enough to increase the water temperature to 20 ºC. But the simulation shows that the temperature never reaches 20 ºC. After 15 minutes, for example, the water temperature is only up to around 19.7 ºC. There is still a temperature difference across the glass surface of 0.3 ºC. This means the heat flowing into the glass is much smaller.

By the 30th minute of the simulation, the heat flowing into the glass has declined almost to the same value as the heat loss from evaporation. The water temperature appears to be approaching 20 ºC after 30 minutes, but it is still not quite the same as the air temperature. By the 45th minute of the simulation, the heat flows in and out of the water are nearly equal. And the water temperature appears to be leveling off slightly below 20 ºC. By the end of the simulation, the water temperature is 19.92 ºC.

We see a small temperature difference across the surface of the glass, and it isn't going away! The size and persistence of this temperature difference is the focus of the exercises.

Exercises with the Model of Water Temperature

1. Verification:

Build the heat flow model in Figure 2 and verify the results in Figure 3.

2. Longer Term Expectations:

What do you expect will happen if we allow the simulation to continue for another hour or two? Will the water temperature eventually reach 20 ºC? Run the model for two hours to see if the simulation results match your expectations.

3. Equilibrium Diagram

If the water temperature reaches equilibrium, draw an equilibrium diagram to confirm that the HeatFlowIn is countered exactly by the HeatLoss.

4. Thinner Glass:

The 0.5 cm thickness is much thicker than typical glass containers, so run the model with the thickness set at 0.25 cm. Before performing the new simulation, pencil in the likely results on a time graph from the 1st exercise. Then run the model with the new value of the thickness. Do the simulation results match your expectations?

5. Derive the Long Term Temperature Difference:

Write an algebraic expression that will permit you to derive the 0.08 ºC as a combination of the physical parameters (such as the glass thickness and conductivity). Check the algebraic expression to see if it gives the same results as in exercises #2 and #4.

6. Experimental Verification with a Standard Thermometer:

A simple glass thermometer might reveal a temperature difference of 0.2 ºC. Suppose you wish to design an experiment where the expected temperature difference is at least 0.2 ºC, and you have glasses of many different thicknesses in your laboratory. How thick must the glass be to yield a measurable temperature gap?

Reference and End Note

Schecker 1996
Horst Schecker, "Modeling Physics: System Dynamics in Physics Education," The Exchange, Vol 5, Nu. 2, the newsletter of the Creative Learning Exchange, 1 Keefe Road, Acton, MA 01720.

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[1] The formula

$$F = m*A$$

tells us the force that must be applied to an object to achieve acceleration of the object. Students learn this formula in introductory physics classes, and they apply it frequently to find the velocity of an object after application of a force. Their familiarity with this formula has led some of my previous students to ask when they get to use the formula in system dynamics modeling. They are surprised to learn that the formula does not come up in the book or on the BWeb materials to supplement the book. It is natural to wonder how one could build useful models and never invoke such an important and fundamental law of physics. It appears from the examples in the book that we can simulate the dynamics of environmental and economic systems without ever invoking the formula F = m*A. In the Mono Lake case, for example, we know that water flows down the Sierra Nevada slopes due to the force of gravity. But we do not need to invoke F = m*A to calculate the annual flow toward Mono Lake. (We have data from gauges on the streams to tell us the annual flow.) To take a different example from the book, commercial buildings must be constructed to withstand the force of gravity, but that does not mean that F = m*A must appear somewhere in the model of the boom and bust in real-estate construction. (The effect of gravity is implicit in the cost of new constructing new buildings.)

The most likely opportunity to make use of F = m*A is the hiking example on the BWeb. The **Let's Go For a Hike Exercise** describes a group of hikers who must apply force to accelerate themselves to a natural pace for the hike up the mountain. The hikers must overcome the force of gravity, and they must deal with complicated issues involving the traction that their hiking shoes achieve on the trail surface. You might think F = m*A would be useful to help us simulate their progress up the mountain. But if you interview typical hikers, you will learn that they have no idea what force is applied as they accelerate themselves to a good pace. Although the hikers cannot talk in the language of introductory physics, they can certainly tell you about their ability to accelerate during the course of the hike. And they can tell you the natural pace that they prefer to maintain over most of the hike. The hiking exercise assumes that you have interviewed the hikers, and you put the interview results to use in the model.