SELECTION & THE SINGLE DIALLELIC DIPLOID LOCUS. 1

• General equations

– Diploid selection to asexual/haploid selection except that parental genotypes not passed on, although their alleles are.

– Will repeatedly use the following facts from our H-W discussion:
  (1) Offspring allele frequencies = those of their randomly mating parents
  (2) Offspring genotype frequencies are in H-W proportions, even if their parents’ were not.

– Relative fitnesses of diploid genotypes: \( w_{AA} , w_{Aa} , w_{aa} \)

– frequency of \( A \) allele = \( p \); frequency of allele \( a = q = 1 - p \).
  • assume zygotes are in H-W proportions.

– Life Cycle: zygotes \( \xrightarrow{\text{survival}} \) adults \( \xrightarrow{\text{fecundity}} \) gametes \( \xrightarrow{\text{random union}} \) zygotes \( \xrightarrow{\text{gen.}\,t+1} \)

– Assume all have same fecundity so relative fitness = viability

– Genotype frequencies after selection (but before reproduction)

  • denoted by \(*\)

  \[
  P_{AA}^* = \frac{\text{number of } AA\text{ parents surviving}}{\text{total number of surviving parents}} = \frac{Np^2w_{AA}}{Np^2w_{AA} + N2pqw_{Aa} + Nq^2w_{aa}} = p^2\frac{w_{AA}}{\bar{w}}
  \]

  where \( \bar{w} = p^2w_{AA} + 2pqw_{Aa} + q^2w_{aa} \) is the population mean fitness.

  • Similarly, \( P_{Aa}^* = 2pq\frac{w_{Aa}}{\bar{w}} \), \( P_{aa}^* = q^2\frac{w_{aa}}{\bar{w}} \).

– Allele frequencies after selection and reproduction

  • If surviving parents mate randomly, then

  \[
  p' = p^* + \frac{1}{2} P_{Aa}^* = p^2\frac{w_{AA}}{\bar{w}} + pq\frac{w_{Aa}}{\bar{w}} = p\left( \frac{pw_{AA} + qw_{Aa}}{\bar{w}} \right)
  \]

  • Quantity in parentheses is mean fitness of individuals who carry \( A \):

   \[
   \bar{w}_A = pw_{AA} + qw_{Aa}
   \]

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Can then rewrite above as, \( p' = p \frac{w_A}{w} \).

Similarly, \( q' = q \frac{w_a}{w} \) where \( w_a = pw_{Aa} + qw_{aa} \) is the mean fitness of individuals who carry \( a \).

- **Handy Fact:** \( \bar{w} = p \bar{w}_A + q \bar{w}_a \) (compare with asexual mean fitness)

- Rate of allele frequency change (\( \Delta p = p' - p \)):
  \[
  \Delta p = pq \frac{w_A - w_a}{w}
  \]

- Summarize:
  - Equations for evolution by selection in diploids identical (in form) to asexuals/haploids.
  - Allele frequency change (evolution) depends on genetic variation and fitness differences

- **Spread of an advantageous allele**

  - Designate \( A \) to be the advantageous allele.

  - Adopt the following notation:

<table>
<thead>
<tr>
<th>Genotype</th>
<th>( AA )</th>
<th>( Aa )</th>
<th>( aa )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>( w_{AA} = 1 )</td>
<td>( w_{Aa} = 1 - hs )</td>
<td>( w_{aa} = 1 - s )</td>
</tr>
</tbody>
</table>

- \( h \) measures allelic dominance of \( A \) over \( a \):
  - \( h = 1 \iff 1:1-s:1-s \) (\( A \) is recessive advantageous)
  - \( h = 0 \iff 1:1:1-s \) (\( A \) is dominant advantageous)
  - \( h = \frac{1}{2} \iff 1:1-s/2:1-s \) (semi-dominant, additive, no dominance)
  - \( h \neq 0,1,\frac{1}{2} \) (partially dominant)

- Equations in this parameterization for allele frequency evolution:
  \[
  \Delta p = pq \frac{\frac{1}{2}q + h(p-q)}{1 - sq(q + 2ph)}
  \]
– Application (from Y.J. Chung. Genetics. 1967)

- Selection against allele that produces stubble bristles in *Drosophila*

<table>
<thead>
<tr>
<th>Genotype</th>
<th>+/+</th>
<th>+/sb</th>
<th>sb/sb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phenotype (Fitness)</td>
<td>normal</td>
<td>stubble</td>
<td>lethal in larval stages ( (s = 1) )</td>
</tr>
</tbody>
</table>

– Two Special Cases:

1. **Dominant** advantageous allele \( h = 0 \), \( W_{AA} : W_{Aa} : W_{aa} = 1 : 1 : 1 - s \)

\[
\Delta p = p(1 - p) \frac{s(1 - p)}{1 - s(1 - p)}
\]

- Dynamics: spread of allele is rapid at first; later, slows as *A* spreads.

- Why these dynamics?
  – Biological Intuition
  – Mathematical Explanation

2. **Recessive** advantageous allele \( h = 1 \), \( W_{AA} : W_{Aa} : W_{aa} = 1 : 1 - s : 1 - s \)

\[
\Delta p = p(1 - p) \frac{sp}{1 - s(1 - p^2)}
\]

- Dynamics: spread of allele *A* is slow at first; rapid later as *A* spreads.

- Why these dynamics?
  – Biological Intuition
  – Mathematical Explanation

– Rates of evolution: Dominant vs. Recessive Alleles

- Basic Principles:
  – rare alleles occur primarily in heterozygotes
– intuition for the magnitude of change when an allele is rare: compare heterozygous fitness to that of the common homozygote.

• Main Features:

  – Extensive time needed for spread of advantageous recessive (extensive time needed for elimination of deleterious recessive)

  – Rapid evolution from 0.1 to 0.9 in all cases

  – Multiplicative selection: produces even faster evolution (homework)

• For weak selection ($s \ll 1$), time required for a specified allele frequency change is inversely proportional to $s$, i.e., $t \propto 1/s$ when $s \ll 1$. 