1. (a) \( \text{rank}(A) = 2. \)
\[
\begin{pmatrix}
  1 & 0 & -2 & 1 \\
  0 & 1 & 1 & 0 \\
  0 & 0 & 0 & 0
\end{pmatrix}
\]
(Underlined entries are pivots.)
(b) \[
\begin{pmatrix}
  1 & 0 & -2 & 1 \\
  0 & 1 & 1 & 0 \\
  0 & 0 & 0 & 0
\end{pmatrix}
\]
(c) Basic columns: \( A_{\ast 1}, A_{\ast 2}. \) Nonbasic columns: \( A_{\ast 3}, A_{\ast 4}. \)
Relationships: \( A_{\ast 3} = -2A_{\ast 1} + A_{\ast 2}, \quad A_{\ast 4} = A_{\ast 1} \)

2. (a) Gaussian elimination will not introduce nonzero elements into a column containing only zeros. Consequently, a column containing all zeros cannot contain a pivot since pivots, by definition, must be nonzero. Thus, a column with all zeros cannot be basic.
(b) Consider the matrix
\[
\begin{pmatrix}
  * & * & * \\
  0 & * & *
\end{pmatrix}
\]
where each * represents a nonzero entry. This matrix is in row echelon form; the pivots are underlined. Although the 3\textsuperscript{rd} column contains no zero entries, it also has no pivot and so is nonbasic.

3. The system is consistent for any values of \( b_1 \) and \( b_2. \)

4. \( A \) is symmetric but not skew-symmetric, hermitian, or skew-hermitian.

5. Conjugate transposition is not linear since \( (\alpha A)^* = \overline{\alpha} A^* \neq \alpha A^*. \)