Barrante 2-1: Equations of the second degree

2-i
\[ x^2 + y^2 = 4 \quad y = \sqrt{4 - x^2} \]

Place the blue editing line on the variable then use "Symbolics" scroll down to "variable" and then select "solve"

\[ x^2 - 4 = 0 \quad \left( \begin{array}{c} 2 \\ -2 \end{array} \right) \]

Finally, open the Symbolic menu and select "solve" and type in the variable name.

\[ q^2 - 4 = 0 \quad \text{solutions for } q \rightarrow \left( \begin{array}{c} 2 \\ -2 \end{array} \right) \]

Roots of a quadratic equation:

\[ 4r^2 + r - 2 = 0 \]

1. "root" function

\[ r := -1 \]

\[ \text{root}(4r^2 + r - 2, r) = -0.843 \]

2. "given ... find " block function

\[ v := -1 \]

\[ \text{Giver} \quad 4v^2 + v - 2 = 0 \]

\[ \text{Find}(v) = -0.843 \]

3. Using "Symbolics", variable and solve

\[ 4x^2 + x - 2 = 0 \]

\[ \left\{ \begin{array}{l} -\frac{1}{8} + \frac{1}{8} = \frac{1}{2} \\ \frac{1}{33} \cdot \frac{1}{8} - \frac{1}{8} = 0.593 \\ -\frac{1}{8} - \frac{1}{8} = -0.843 \end{array} \right\} \]
4. Using "solve, x"

\[ 4y^2 + y - 2 = 0 \]

\[ \text{solves for } y \rightarrow \left( \frac{-1 + \frac{1}{8} \cdot 0.33^2}{8}, \frac{1}{8}, \frac{-1 - \frac{1}{8} \cdot 0.33^2}{8} \right) \]

5. High School Algebra:

It has the form \( ax^2 + bx + c = 0 \) and the roots are \( \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

\[
\begin{align*}
a &:= 4 \\
b &:= 1 \\
c &:= -2 \\
r1 &:= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\
r2 &:= \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\end{align*}
\]

\[
\begin{align*}
\text{r1} &= 0.593 \\
\text{r2} &= -0.843 \\
2-j \\
\end{align*}
\]

\[
(\text{x1} - 2)^2 + (\text{y1} + 4)^2 = 9
\]

\[
f(\text{x1}) := (\text{x1} - 2)^2 + 4^2 - 9
\]

\[
\text{x1} := 2i \\
\text{root}\left[(\text{x1} - 2)^2 + 4^2 - 9, \text{x1}\right] = 2 + 2.646i \\
\text{OR} \\
\text{root}(f(\text{x1}), \text{x1}) = 2 + 2.646i
\]

**Solving a Nonlinear System of Equations**

Solving a system of \( n \) equations in \( n \) unknowns using "Given .... Find" solve block.

Enter guess values for the \( n \) unknowns:

\[
\begin{align*}
\text{x2} &:= 1 \\
\text{y2} &:= -1 \\
\text{z2} &:= 0
\end{align*}
\]

Enter the \( n \) equations:

Given

1. \( 2 \cdot \text{x2}^3 - 2 \cdot \text{y2} = 7 - 2 \cdot \text{z2}^4 \)

2. \( \text{y2}^3 + 4 \cdot \text{z2}^2 = 4 \)

The symbolic equals sign [Ctrl]= should be used to define equations within the Solve Block.
3. \[ x_2 y_2 + z_2 \cdot \ln(x_2) = e^{z_2} \]

\[
\text{soln} := \text{Find}(x_2, y_2, z_2) \\
\text{soln} = \begin{pmatrix} 1.454 \\ 0.51 \\ -0.983 \end{pmatrix}
\]

Barrante 4.1(d, l, n, r)
Differentiate the following functions.
(All the lowercase letters are the variable and All uppercase letters are constants.)

(d) \[ r = 3 \cdot \tan(2 \cdot \theta) \]

\[ 0 = 6 + 6 \cdot \tan(2 \cdot \theta)^2 \]

(l) \[ s = \ln(t) \cdot e^{-3 \cdot t} \]

\[ 0 = \frac{1}{t} \cdot \exp(-3 \cdot t) - 3 \cdot \ln(t) \cdot \exp(-3 \cdot t) \]

(r) \[ u = \frac{A}{r^{12}} - \frac{B}{r^6} \]

\[ 0 = -12 \cdot \frac{A}{r^{13}} + 6 \cdot \frac{B}{r^7} \]

Finally:
\[ d = 3 \cdot \cos(\theta)^2 - 1 \]

\[ h = \sin(\theta) \]

\[ 0 = \cos(\theta) \]

---

**Derivatives and Integrals**

**Numerical evaluation of the derivative of a function for a set of values.**

Enter a function \( h(x) \) you want to differentiate:

\[ h(x) := 13 \cdot x^7 \]

Enter points at which to compute derivative (as a range variable):

\[ i := 0 .. 4 \]

\[
\begin{array}{c|c|c|c}
\hline
i & h(i) & \frac{dh}{dx}(i) & g(i) \\
\hline
0 & 0 & 0 & 0 \\
13 & 1664 & 91 & 4 \\
28431 & 66339 & 16 & 6 \\
212992 & 372736 & 8 & \\
\hline
\end{array}
\]
Enter points at which to compute derivative (as a vector):

Be sure to type “1i” rather than “i” for the imaginary unit.

\[
\begin{pmatrix}
3 \\
2 + i \\
e^{0.3 \cdot \pi} \\
0.223 \\
0.125
\end{pmatrix}
\]

\[
\begin{pmatrix}
-3614 - 377i \\
9530.895 \\
0 \\
0
\end{pmatrix}
\]

\[
\begin{pmatrix}
66339 \\
-10647 + 4004i \\
25996.736 \\
0.011 \\
0
\end{pmatrix}
\]

**Some use of Symbolic function:**

Limit: \( \lim_{x \to 0} \frac{1}{(1 + x)^x} \to e \) use of “------” from the Symbolic menu

Derivative: \( \frac{d^2}{dx^2} \cos(x)^3 \to 6 \cdot \cos(1) \cdot \sin(1)^2 + 3 \cdot \cos(1)^3 \) use of “------” from the Symbolic menu

Integral: \( \int_a^b x^2 \, dx \to -21 \)

**Finite integral:**

\[
\int_0^{10} x^2 \, dx = 333.333 \\
\int_0^1 x^3 + \sin(x) \, dx = 0.71
\]

Since \( a \) below is constrained to be positive, this improper integral will converge. With unconstrained \( b \), a more complicated expression emerges.
Hints for Homework#3

For problem 2 use the fitting function "pspline" and "interp".

Use the function "spline" for the interpolation:
3 types of splines: 1. cspline: cubic interpolation, the resultant spline curve is cubic at the end points.
2. pspline: parabolic at the end points.
3. lspline: linear at the end points

Note: When you have the data rather than the formula there are 2 ways to find the formula
1. use curve fitting.
2. interpolate the data (Mathcad forte), spline method used with interp

cspline and interp functions for connecting X-Y data.

Enter a matrix of X-Y data to be interpolated:

\[
\begin{array}{c|c|c}
  & 0 & 1 \\
\hline
0 & 0 & 0 \\
1 & 1 & 3 \\
2 & 4 & 27.57 \\
3 & 5 & 25 \\
4 & 7 & 18 \\
5 & 8 & 30.41 \\
6 & 11 & 48 \\
7 & 13 & 62 \\
8 & 14 & 59.89 \\
9 & 16 & 72.28 \\
10 & 17 & 83 \\
11 & 18 & 65 \\
12 & 19 & 78 \\
13 & 20 & 85 \\
\end{array}
\]

\[M := \text{csort}(M, 0)\]
\[X := M^{(0)}\]
\[Y := M^{(1)}\]

Spline coefficients:
\[S1 := \text{cspline}(X, Y)\]

Fitting function:
\[\text{fit}(x) := \text{interp}(S1, X, Y, x)\]

Sample interpolated values:
For problem 3 use "root" after using "pspline and interp" to calculate $\Delta H$, remember you have to guess a value for $T$. 

\[ f(3) = 22.023 \quad f(9.5) = 41.232 \]