General Expression for the internal energy $U$ and the enthalpy $H$

\[ dU = C_v \, dT + \left[ T \cdot \left( \frac{\partial P}{\partial T} \right) - P \right] \, dV \]

\[ dH = C_p \, dT + \left[ V - T \cdot \left( \frac{\partial V}{\partial T} \right) \right] \, dP \]

Special cases:
1) ideal gases
2) constant volume and pressure processes

1. Ideal gases: the second term on both expressions is zero

\[ dU = C_v \, dT \]

\[ dH = C_p \, dT \]

2. Constant volume the second term in $dU$ equals zero and at constant pressure the second term in $dH$ equals zero

\[ dU = C_v \, dT \]

\[ dH = C_p \, dT \]

To calculate the enthalpy from the heat capacity at constant pressure $C_p$ you need to integrate the expression of $C_p$ in the range of temperatures.

\[ \Delta H = \int_{T_1}^{T_2} C_p(T) \, dT \]

Finally for real gases the internal energy $U$ has two terms; the first term is $U_0$ (standard internal energy) and the second term is called the imperfection internal energy $U_i$. This latter term is given by

\[ U_i = \int_{\infty}^{V} \left[ T \cdot \left( \frac{\partial P}{\partial T} \right) - P \right] \, dV \]

If you use the Redlich-Kwong expression for $P$ in the following form and use Symbolics and the differentiate you will get
\[ P(V, T) = \frac{R \cdot T}{V - b} - \frac{a}{\sqrt{T} \cdot V(V - b)} \]

\[ \frac{d}{dT} P(V, T) = \frac{R}{V - b} + \frac{1}{2} \cdot \frac{a}{\frac{3}{2} T^2 \cdot V(V - b)} \]

then \( U_i \) is given by

\[
U_i = \left[ \frac{R}{V - b} + \frac{1}{2} \cdot \frac{a}{\frac{3}{2} T^2 \cdot V(V - b)} \right] - P \int_{\infty}^{V} dV
\]

and the final equation for \( U_i \) is

\[
U_i(V, T) = \frac{3}{2} \cdot \frac{a}{\sqrt{T} \cdot b} \cdot \ln \left( \frac{V}{V + b} \right)
\]