Error Analysis for Laboratory Reports

There are **three steps** in error analysis of most experiments.

1. **Propagation of errors** can be performed even before the experiment is performed.
2. **Measuring the errors** is done during the experiment.
3. **Comparison with accepted values** is performed after the experiment is completed.

This means a **numerical** error analysis, **not** a post-experimental accounting of what you could do to improve your results.

Available source of information on error analysis is **in Chapter 2 of your laboratory textbook**.

Your laboratory report needs to include the following:

- For three or more point repetitions, do the average and standard deviation for each point (see chapter 2 of the textbook or a quantitative analysis text).
- Propagate measurement errors through calculations.
- Report all calculated values with error and units attached, e.g. 4.2 ± 0.1 mL.
- Compare results with accepted and/or literature values

**Example of Propagation of Errors**

One of the experiments involves measuring the heat capacity ratio for a certain gas, say Argon. Two equations are used in this experiment:

\[
c = \nu \lambda \quad \text{and} \quad \gamma = \frac{M c_v}{RT} \quad \text{where } \gamma \text{ is the heat capacity ratio, } \frac{C_p}{C_v}.
\]

I. The variables are \( \nu, \lambda, \) and \( T \).

1. Estimate the error in the measurement of these three quantities. To do this you must decide how accurate the measurement was to within a certain number of units and you must be realistic.

2. Average values ± uncertainties

\[
\nu = 2207 \pm 2 \text{Hz},
\]
\[
\lambda = 144.8 \pm 0.5 \text{ mm}
\]
\[
T = 296.8 \pm 0.2 \text{ K}
\]

Since we must use the measured values to find \( c \) first, we must first propagate through that equation to find the error in the calculated value of \( c \).

Use equation 52 on page 58 of the text. In this case, \( F(x,y) = F(\nu,\lambda) = c = \nu \lambda \).

\[
\Delta^2 F = \left[ \left( \frac{\partial F}{\partial x} \right)^2 \Delta^2 x + \left( \frac{\partial F}{\partial y} \right)^2 \Delta^2 y \right]
\]

Substituting \( c \) in for \( F \) gives an equation of the form:
\[ \Delta^2 c = \left( \frac{\partial c}{\partial \nu} \right)^2 \Delta^2 \nu + \left( \frac{\partial c}{\partial \lambda} \right)^2 \Delta^2 \lambda \]

After the actual derivations:

\[ \Delta^2 c = \lambda^2 \Delta^2 \nu + \nu^2 \Delta^2 \lambda \]

Plugging the numbers from above into this equation gives

\[ \Delta^2 c = (144.8)^2 (2)^2 + (2207)^2 (0.5)^2 = 83,868.2 + 1,217,712.2 = 1,301,580.4 \]

and

\[ \Delta c = 1140.8 \]

The result should be written as \( c = 319.5 \pm 1.1 \text{ m/sec} \)

II. Value of \( \gamma \) and its propagated error.

\[ \gamma = \frac{Mc^2}{RT} = \frac{0.039948(319.5)^2}{8.324(296.8)} = 1.65 \]

\( M_{Ar} = 39.948 \text{ g/mol} = 0.039948 \text{ Kg mol}^{-1} \)

Gas constant \( R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \)

\( T = 296.8 \pm 0.2 \text{ K} \) and \( c = 319.5 \pm 1.1 \text{ m/sec} \)

The propagation equation will look like:

\[ \Delta^2 \gamma = \left[ \left( \frac{\partial \gamma}{\partial c} \right)^2 \Delta^2 c + \left( \frac{\partial \gamma}{\partial T} \right)^2 \Delta^2 T \right] = \left( \frac{2Mc}{RT} \right)^2 \Delta^2 c + \left( \frac{-1Mc^2}{RT^2} \right)^2 \Delta^2 T \]

Again plugging in the values from above into the equation \( \Delta^2 \gamma \) becomes:

\[ \Delta^2 \gamma = \left( \frac{2\times(0.039948)\times319.5}{8.314\times(296.8)} \right)^2 (1.1)^2 + \left( \frac{-0.039948\times(319.5)^2}{8.314\times(296.8)^2} \right)^2 (0.2)^2 = 1.29 \times 10^{-4} + 1.24 \times 10^{-6} \]

\( \Delta \gamma = 0.01 \) and then the value for the heat capacity ratio with its error is then

\[ \gamma = 1.65 \pm 0.01 \]

Note that from the above propagation of error one could point out which quantity contributes the most to the error. The first term \( (1.29 \times 10^{-4}) \) is 2 orders of magnitude larger than the second term. By improving the way \( c \) is measured the error on \( \gamma \) could be reduced.